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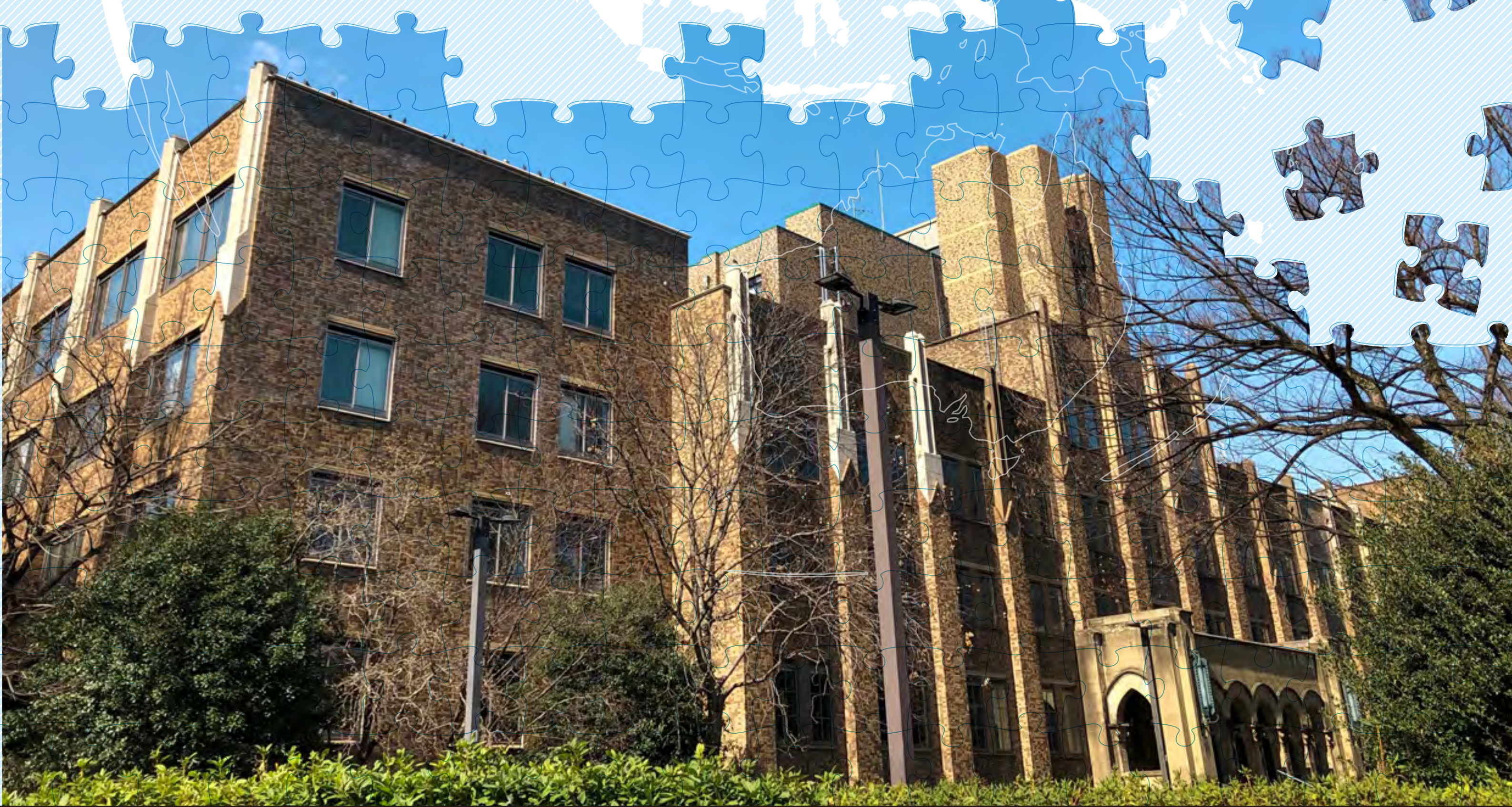
Economic Development and Fertility Rebound



No. 125

Date August.2025

SDGs

8 DECENT WORK AND ECONOMIC GROWTH

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Economic Development and Fertility Rebound

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August 6, 2025

Abstract

In this article, we examine a mechanism by which the fertility rebound occurs within the context of an overlapping-generations model with uncertain lifetime. We segment the stages of economic development into early, middle, and long-term stages. Additionally, we analyze the influence of life expectancy, the elasticity of substitution, and the gender wage gap on the fertility rate at each stage of economic development. The key results are as follows: In the early stage, the fertility rate is independent of the three factors mentioned above. Improved life expectancy implies a decrease in the fertility rate in the middle stage. This result is the same in the case in which the economy is in the long-term stage. The fertility rate when the elasticity of substitution indicates a high value is larger than that when the elasticity of substitution indicates a low value in the middle stage. However, when the economy is in the long-term stage, an increase in the elasticity of substitution implies an increase in the fertility rate. A decrease in the gender wage gap means a decline in the fertility rate when the economy is in the middle stage. This implication is the same as that in the long-term stage.

Keywords: Couples' decision-making problems, Fertility rebound, Gender wage gap, Life expectancy at birth, Sustainable development goals

JEL Classification: D10, J11, J13, J14, O11

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1 Introduction

Total fertility rates (TFRs) in many Organisation for Economic Co-operation and Development (OECD) member countries declined continuously from the mid-1980s to the early 2000s (see, e.g., Luci-Greulich and Théevenon, 2013; Day, 2018; Ohinata and Varvarigos, 2020). Based on Becker's (1960) child quality–quantity model, such a decline in TFRs can be explained by the increasing importance of investment in the quality rather than the quantity of children as economic development progresses. An influential study following the seminal work of Becker (1960) was conducted by Galor and Weil (1996). According to Galor and Weil (1996), as economic development narrows the gender wage gap, the opportunity cost of childcare increases. As a result, women's employment rates increase and TFRs decline. However, in several recent empirical studies, researchers observed a positive correlation between TFRs and indicators of economic development (e.g., gross domestic product (GDP) per capita, human development index (HDI), and female labor force participation) in several advanced economies that have been members of the OECD since the mid-2000s (Myrskylä et al., 2009; Lacalle-Calderon, 2013: Luci-Greulich and Théevenon. 2014; Nakagaki, 2019; Mavropoulos and Panagiotidis, 2021).[1] Such a phenomenon is often called the fertility rebound.

In this article, we have two objectives. The first objective is to explain a mechanism by which the so-called fertility rebound occurs from a theoretical perspective in the context of an overlapping-generations (OLG) model with an uncertain lifetime. In our model, we classify the process of economic development into three stages as follows: early stage, middle stage, and long-term stage. In particular, the fertility rebound may occur in the long-term stage of economic development. The second objective is to clarify the effects of life expectancy at birth, the elasticity of substitution between capital and labor, and the gender wage gap on the fertility rate at each stage of economic development. Furthermore, we present policy implications based on our analytical results.

For other attempts to explain the mechanism of the fertility rebound using theoretical approaches, representative studies include those conducted by Day (2018), Yakita (2018), and Ohinata and Varvarigos (2020). For example, Day (2018) stated that, through an increase in women's wages relative to men's wages caused by economic growth, mothers are able to use childcare services instead of using their own time to raise children, and thus couples have more children;[2] that is, Day (2018) showed that the fertility rate could

[1]The HDI is the key index used by the United Nations Development Programme to monitor and evaluate broadly defined human development, combined with equal weight indicators of a country's health conditions, standard of living, and human capital (see, Myrskylä et al., 2009, for details).

[2]The econometric research of Hazan and Zoabi (2015) is related to Day's (2018) discussion and partially complements the results. Hazan and Zoabi (2015) examined the relationship between women's educational level and the fertility rate in the United States using micro data. Their main results are as follows: Until the 1990s, highly educated women had relatively few children in the United States. However, in the 2000s, highly educated women who had high incomes had higher fertility rates than women with moderate levels of education. According to Hazan and Zoabi (2015), in addition to an increase in the number of women's groups with the financial means to use housework and childcare services, there was also an increase in the number of women's groups willing to provide such services at a relatively low cost. This evidence is considered to be the main reason that the number of births among highly educated women with high incomes has increased.

shift from a decline to a rise because of an increase in women's wages caused by economic growth. In Day's (2018) model, the logarithm of GDP per capita depends on factors such as subsidized childcare, maternity allowances, and the value of time spent raising children. In addition, when the logarithm of GDP per capita increases beyond a certain threshold, the fertility rate changes from a decline to a rise. Day (2018) considered two aspects of couples' decision-making problems. The first is the amount of childcare services couples purchase given the mother's time and the number of children. The second is choosing the number of children that maximizes utility under the budget constraint. Day (2018) stated that her model that explains the mechanism of the fertility rebound is robust even when assumptions are relaxed about the utility function. However, it should be noted that Day's (2018) model is essentially a static model, not a dynamic model.

Like Day (2018), Yakita (2018) also considered the possibility of the fertility rebound by focusing on the relationship between the women's wage rate and the price of childcare services such as daycare centers and kindergartens. However, unlike Day (2018), Yakita's (2018) model takes the form of an OLG model, which is a model of definite macroeconomic dynamics, and is constructed by combining the ideas of Galor and Weil (1996) and Kemnitz and Thum (2015). Regarding the characteristics of Yakita's (2018) model, the process of economic development consists of three stages, as in our model, and the fertility rebound occurs under certain conditions, such as when children are normal goods.[3] An important implication of the model for the mechanism of the fertility rebound is that an increase in women's wages associated with economic development makes external childcare services more available; that is, as a precondition that the fertility rebound occurs, women's wages must exceed the price of such childcare services. Consequently, the fertility rebound occurs endogenously without policy intervention by the government.

Ohinata and Varvarigos (2020) proposed a theory to explain the dynamics of the fertility rate using a two-period OLG model. According to their model, at the initial stage of industrialization, the level of GDP is low and hence, the fertility rate is low. Subsequently, as industrialization develops, the fertility rate also increases. However, in the intermediate stage, when the economy becomes affluent, to a certain extent, parents encounter a trade-off between the quantity and quality of their children; that is, a demographic transition with a decline in fertility appears at an intermediate stage because of the consideration of the human capital investment in children. Moreover, in a later stage, economic growth allows couples to support more children while still providing the desired amount of educational investment for each child. Thus, Ohinata and Varvarigos (2020) showed that the shape of such fertility rate movement is an N-shaped curve and explained the mechanism by which the fertility rebound occurs.

[3] Black et al. (2013) examined cross-sectional data for non-Hispanic white couples in the United States and showed that fertility is positively correlated with the husband's income when the comparison is limited to women with similar levels of education and living in similarly expensive locations. This empirical evidence is consistent with children being normal goods. Moreover, Day and Guest (2016) considered the scenario in which housing costs occupy an important position for child rearing among the various costs of child rearing. The results of the empirical analysis based on the theoretical model suggested that if the importance of housing costs is sufficiently high relative to other costs for child rearing, as described above, the negative substitution effect on fertility when women's wages increase is weakened and fertility may increase with an increase in women's wages.

Among the studies mentioned above, our work is most closely related to that of Yakita (2018); however, our work has the following differences from Yakita's (2018) work in terms of the model settings and analytical approach. Specifically, Yakita's (2018) OLG model assumed that all individuals certainly lived for two periods. This assumption means that people's life expectancy at birth is always fixed at a constant value. In this regard, we look at the historical fact that, as shown by Weil (2013, Ch.4), life expectancy in many countries has continued to increase consistently since the 19th century. We believe that such evidence cannot be ignored when considering the issue of the fertility rebound. Therefore, to analyze the effects of an improvement in life expectancy at birth (i.e., an increase in longevity) on the fertility rate in each phase of economic development, we need to extend Yakita's (2018) OLG model by introducing the survival probability in old age. This is the reason that we mentioned that this study uses the OLG model with an uncertain lifetime.

Moreover, we replace the part of the formulation of a general production function in the consumer goods production sector in Yakita's (2018) model with a constant elasticity of substitution (CES) production function. This specification of the production function allows us to analyze how the elasticity of substitution between capital and labor influences the fertility rate. The reason for considering the elasticity of substitution in this study is as follows: Nakamura (2009) provided an interesting insight into the relationship between the elasticity of substitution and an increase in the range of capital use as a form of mechanization. In Nakamura's (2009) model, an increase in the range of capital use means that capital is used more extensively than labor. Based on such an interpretation, Nakamura analyzed how mechanization is related to the elasticity of substitution between capital and labor. The major contributions are that Nakamura (2009) presented a microeconomic foundation for a CES production function based on the concept of mechanization and clarified the relationship between the magnitude of the elasticity of substitution and the degree of difficulty of mechanization in the context of an endogenous growth model. For example, in the case in which the value of the substitution parameter is positive, when the substitution parameter is relatively large (i.e., the elasticity of substitution is relatively small), mechanization becomes more difficult. Conversely, when the substitution parameter is relatively small (i.e., the elasticity of substitution is relatively large), mechanization becomes easier. Thus, Nakamura's (2009) model provides a basis for the varying degree of difficulty of mechanization across industries and economies in terms of differences in the elasticity of substitution between capital and labor. Clearly, increasing mechanization is a quintessential feature of firms' production activities in a global economy. Hence, examining the effect of the elasticity of substitution on the fertility rate is a key issue because it is equivalent to comprehending the degree of economy-wide mechanization in a country on the level of the fertility rate.

Furthermore, we perform comparative statics with respect to the influences of life expectancy at birth, the elasticity of substitution, and the gender wage gap on the fertility rate. In this regard, we use a useful method of numerical analysis to improve our understanding of the implications of the model and achieve our research objectives. Specifically, when we perform comparative statics at each phase of economic development, the designated parameter changes to the value indicated, whereas all other parameters remain at their baseline settings. Using this method of numerical analysis, we can visually capture the features of the impact of the above three factors on the fertility rate by graphing the change in fertility with capital accumulation per couple. Based on the results of this quantitative approach to comparative statics, we also perform policy analysis. To the best of our knowledge, such a quantitative approach to comparative statics and policy analysis does not exist in related studies. Consequently, our proposed quantitative approach allows us to obtain important findings that have been overlooked to date and contributes to the development of the research field of couples' decision-making problems in a broad sense (the fertility rebound in a narrow sense).

The main results of our study are as follows: In the early stage of economic development, the fertility rate is not affected by life expectancy at birth, the elasticity of substitution, and the gender wage gap because it becomes a constant. Therefore, our analytical results are focused on the middle and long-term stages of economic development. First, regarding the effects of life expectancy at birth on the fertility rate, an improvement in life expectancy at birth implies a decrease in the fertility rate in the middle stage of economic development. In addition, an improvement in life expectancy at birth leads to an increase in the rate of the population of older adults. Thus, our model suggests that the progress of population aging would result in a decrease in the fertility rate in the future. The above results are essentially the same in the case in which the economy is in the long-term stage. Second, regarding the effects of the elasticity of substitution between capital and labor on the fertility rate, the fertility rate corresponding to a large value of the elasticity of substitution is larger than that corresponding to a small value of the elasticity of substitution in the middle stage of economic development. However, when the economy is in the long-term stage, an increase in the elasticity of substitution leads to an increase in the fertility rate. Third, regarding the effects of the gender wage gap on the fertility rate, an decrease in the gender wage gap means a decline in the fertility rate when the economy is in the middle stage of economic development. This implication of our model is essentially the same in the long-term stage of economic development; that is, even when the economy is in the long-term stage, a narrowing of the gender wage gap results in a decrease in the fertility rate. Moreover, our study is related to target 8.1 of goal 8 ("Decent Work and Economic Growth") of the sustainable development goals (SDGs). Specifically, the analytical framework based on our OLG model provides part of the theoretical basis for improving the understanding and achievement of this SDG target.

The remainder of this article is organized as follows: In Section 2, we explain the basic setup of the model. Moreover, we examine couples' decision-making problems. Furthermore, we analyze the profit maximization problems of firms in the consumer goods production sector and childcare services sector. In Section 3, we investigate the relationship between capital per couple and the fertility rate in general equilibrium. In Section 4, we consider the effects of changes in life expectancy at birth, the elasticity of substitution, and the gender wage gap on capital per couple and the fertility rate through comparative statics from a quantitative perspective. In Section 5, we present the policy implications of our model. Finally, in Section 6, we describe the contributions of our research, summarize the main results, and conclude the article.

2 The Model

2.1 Basic Setup

We begin by describing the basic structure of our OLG model with an uncertain lifetime. Suppose that at the beginning of period t, there are N_t couples of men and women. All couples can survive for at most two periods. For convenience, the first period of a couple's life is referred to as the young working period and the second period is referred to as the old retired period. The cohort of individuals born at the beginning of period t is called generation t. Couples in generation $t \geq 0$ definitely live through their young working period. However, survival into the old retired period is uncertain. Following Yakita (2008), Maebayashi (2013), and Watanabe (2015), we assume that a couple in generation $t \geq 0$ survives into the old retired period with probability $\theta \in (0, 1)$, but dies at the beginning of the old retired period with probability $1 - \theta$. Therefore, an increase in θ can be interpreted as an increase in life expectancy at birth (i.e., an extension of longevity); that is, large θ means that more older people live in an older retired period compared with when θ is low. For convenience, we regard θ as a parameter.

We consider a closed economy that consists of households, a consumer goods production sector, and a childcare services sector. In our model, there are differences between men and women with respect to their ability to work. Specifically, men only have the ability to work in the consumer goods production sector. By contrast, women have the ability to work in both the consumer goods production sector and childcare services sector. Thus, a gender wage gap exists because of such differences in work capabilities between men and women.

2.2 Couples' Decision-Making Problems

In this section, we examine couples' decision-making problems. Each household consists of a husband and wife, which are a male and female pair. Because there exist N_t couples at the beginning of period t, the number of households is N_t. Similar to Yakita (2018), to avoid matching problems in marriage, we assume that the number of women and men is the same, and that each couple has the same number of boys and girls. In addition, the utilities that each couple obtain from consumption and the benefits of having children are common.

The time endowment of each period is normalized to 1. In the old retired period, both men and women do not engage in labor. Therefore, the total amount of principal and interest earned on savings during the young working period is used for consumption during the old retired period. In the young working period, men supply one unit of labor service inelastically. By contrast, women encounter the problem of how to allocate time regarding childcare at home and work at firms.

We denote the utility of a couple of generation t by u_t and formulate the utility function as follows:

$$u_t = \log c_t^1 + \varepsilon \log n_t + \rho\theta \log c_{t+1}^2, \tag{1}$$

where c_t^1 represents the consumption of the generation t couple/family in the first period, c_{t+1}^2 represents the consumption of the generation t couple/family in the second period, and n_t denotes the number of children the couple have (more precisely, the same number of boy/girl pairs). Moreover, the coefficient of $\log n_t$ in Eq. (1), $\varepsilon > 0$, measures the weight of the effect of having children on the utility of a couple, $\theta \in (0, 1)$ is the probability that the couple is alive at the beginning of the old retired period, and $\rho \in (0, 1)$ is the subjective discount factor.

The budget constraint of a couple in its young working period is given by

$$w_t^m + (1 - h_t)w_t^f = c_t^1 + P_t x_t + s_t, \tag{2}$$

where P_t is the price of childcare services outside the home, w_t^m is the wage rate of the male, w_t^f is the wage rate of the female, $h_t \in [0, 1]$ is the time the wife spends raising children at home, $x_t \geq 0$ is the amount of external childcare services (usage measured by time), and s_t represents savings.

Following Yakita (2008), Maebayashi (2013), and Watanabe et al. (2015), we assume the existence of risk-neutral insurance companies and a competitive private annuity market. In this case, if the gross interest rate in period t is expressed as r_t, the rate of return on the pension received by a couple of generation t that survives into the old retired period is r_{t+1}/θ. We also assume that all couples leave no inheritance when they die. Therefore, the budget constraint in the old retired period of the couple is given by

$$\frac{r_{t+1}}{\theta} s_t = c_{t+1}^2. \tag{3}$$

Couples raise their children using the wife's time and external childcare services. The husband does not spend time raising children. Let $1/\phi$ be the time required to raise a pair of children, a boy and a girl. Thus, the time required to raise n_t pairs of children is given by $n_t/\phi = h_t + x_t$.

When we represent the cost of raising n_t pairs of children by $C(n_t)$, the following equation holds:

$$\begin{aligned} C(n_t) &= w_t^f h_t + P_t x_t \\ &= (w_t^f - P_t)h_t + P_t \frac{n_t}{\phi}. \end{aligned} \tag{4}$$

Given n_t, the couple chooses h_t and x_t that minimize $C(n_t)$ in Eq. (4). As a result, the minimized childcare costs are expressed as follows:

$$C(n_t) = \begin{cases} \text{(i)} \ \dfrac{P_t n_t}{\phi} & (w_t^f > P_t), \\[2ex] \text{(ii)} \ \dfrac{n_t}{\phi} w_t^f & \left(w_t^f \leq P_t \text{ and } \dfrac{n_t}{\phi} < 1\right), \\[2ex] \text{(iii)} \ w_t^f + P_t\left(\dfrac{n_t}{\phi} - 1\right) & \left(w_t^f \leq P_t \text{ and } \dfrac{n_t}{\phi} \geq 1\right). \end{cases} \tag{5}$$

In case (i) of Eq. (5), the wife does not do any childcare at home and uses all of her time to work at the firm. Therefore, childcare at this time is completely entrusted to external childcare services. Therefore, in case (i), the couple chooses $h_t = 0$, and the external childcare time is represented by $x_t = n_t/\phi$. In case (ii) of Eq. (5), because the couple does not use external childcare services, $x_t = 0$ holds. In addition, the wife spends part of her time raising children at home and the remainder of her time working; that is, the wife's childcare time at home is n_t/ϕ and the wife's working hours are $1 - n_t/\phi$. Finally, in case (iii) of Eq. (5), the wife uses all of her time to raise children at home and does not use any of her time to work at the firm; that is, in this case, $h_t = 1$ holds. In addition, the couple chooses $x_t = 0$ and does not use external childcare services.

Next, we consider the couple's utility maximization problem. From Eqs. (2), (3), and (5), the intertemporal budget constraint can be expressed as follows:

$$\begin{aligned} w_t^m + w_t^f &= c_t^1 + s_t + C(n_t) \\ &= c_t^1 + \frac{\theta c_{t+1}^2}{r_{t+1}} + C(n_t). \end{aligned} \tag{6}$$

First, we consider case (i). Under the conditions in Eqs. (5) and (6), when a couple maximizes its utility, we obtain

$$c_t^1 = \frac{P_t n_t}{\phi \varepsilon}, \tag{7}$$

$$c_{t+1}^2 = \frac{\rho P_t n_t}{\phi \varepsilon} r_{t+1}. \tag{8}$$

After some manipulation, the substitution of Eqs. (7) and (8) into Eq. (6), yields

$$n_t = \frac{\varepsilon \phi}{1 + \varepsilon + \rho \theta} \frac{w_t^f + w_t^m}{P_t}. \tag{9}$$

Moreover, considering Eqs. (3) and (9),

$$s_t = \frac{\rho \theta}{1 + \varepsilon + \rho \theta} (w_t^f + w_t^m). \tag{10}$$

Eq. (10) represents the relationship between the wages for a man and woman, and savings in case (i).

Next, we consider case (ii). Solving the couple's utility maximization problem, we obtain

$$c_t^1 = \frac{w_t^f n_t}{\phi \varepsilon}, \tag{11}$$

$$c_{t+1}^2 = \frac{\rho w_t^f n_t}{\phi \varepsilon} r_{t+1}. \tag{12}$$

The substitution of Eqs. (11) and (12) into Eq. (6) yields

$$n_t = \frac{\varepsilon \phi}{1 + \varepsilon + \rho \theta} \frac{w_t^f + w_t^m}{w_t^f}. \tag{13}$$

Moreover, from Eqs. (3) and (13), we obtain

$$s_t = \frac{\rho\theta}{1+\varepsilon+\rho\theta}(w_t^f + w_t^m). \tag{14}$$

Note that, to satisfy the condition $n_t/\phi < 1$ with respect to Eq. (13), the following relationship must hold in case (ii):

$$\frac{w_t^f}{w_t^m} > \frac{\varepsilon}{1+\rho\theta}. \tag{15}$$

For example, when the weight of the utility of having children is sufficiently small and the level of the subjective discount factor and the survival probability at the beginning of the old retired period are sufficiently high, the relationship in Eq. (15) may hold.

Regarding $w_t^f < P_t$ and $n_t/\phi \geq 1$, if the optimal plans are interior solutions, then we obtain

$$c_t^1 = \frac{P_t n_t}{\phi\varepsilon}, \tag{16}$$

$$c_{t+1}^2 = \frac{\rho P_t n_t}{\phi\varepsilon} r_{t+1}. \tag{17}$$

Considering Eq. (5) and substituting Eqs. (16) and (17) into Eq. (6), we obtain

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho\theta}\frac{P_t + w_t^m}{P_t}. \tag{18}$$

In case (iii), when we note that $w_t^m = c_t^1 + s_t$ holds for Eq. (2) and consider Eq. (18), we obtain

$$s_t = \frac{\rho\theta}{1+\rho\theta} w_t^m. \tag{19}$$

Because n_t must satisfy $n_t/\phi \geq 1$, we obtain

$$\frac{w_t^f}{w_t^m} \leq \frac{\varepsilon}{1+\rho\theta}. \tag{20}$$

In our model, from the assumption in the model that men do not participate in childcare and are always engaged in full-time work, the corner solution $n_t/\phi = h_t = 1$ holds in case (iii); that is, when the relationship in Eq. (20) holds, we obtain

$$n_t = \phi. \tag{21}$$

Therefore, from Eq. (21), in the phase of case (iii), the fertility rate (n_t) does not depend on capital per couple (k_t) and is a constant value.

2.3 Consumer Goods Production Sector

Now we examine the profit maximization problem of firms in the consumer goods production sector. We assume that only one type of consumer good exists in this economy. Such homogeneous consumer goods are regarded as the numéraire and their prices are normalized to 1. The production of Y_t units of consumer goods requires inputs of physical capital K_t, male's labor L_t^m, and female's non-physical (mental) labor L_t^{fY}. Following Galor and Weil (1996), the production function for the consumer goods sector is formulated as follows:

$$Y_t = \left[\delta K_t^{-\eta} + (1-\delta)L_t^{-\eta}\right]^{-\frac{1}{\eta}} + bL_t^m. \tag{22}$$

Note that the first term on the right-hand side of Eq. (22) is a type of CES production function, where $L_t \equiv L_t^m + L_t^{fY}$, $\delta \in (0,1)$ is the distribution parameter, $\eta \in (-1,\infty)$ is the substitution parameter, and $b > 0$ is a parameter that represents the marginal productivity of the male's physical labor. In this case, when the elasticity of substitution between capital (K_t) and aggregate labor (L_t) in the consumer goods sector is expressed as σ, we find that $\sigma = 1/(1+\eta)$ holds.

We define $k_t \equiv K_t/N_t$ and $\ell_t^{fY} \equiv L_t^{fY}/N_t$. Note that $L_t^m/N_t = 1$. We consider the subjective equilibrium of firms in the consumer goods production sector. When firms adopt profit maximizing behavior, the following relationship holds.

$$r_t = \delta\,(k_t)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(1+\ell_t^{fY}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}. \tag{23}$$

From Eq. (23), the gross interest rate (r_t) depends on capital per couple (k_t) and the non-physical (i.e., mental) labor of the woman per couple (ℓ_t^{fY}). Regarding the wage rate for women, a subjective equilibrium condition of firms implies that

$$w_t^f = (1-\delta)\left(1+\ell_t^{fY}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(1+\ell_t^{fY}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}. \tag{24}$$

Moreover, because the firm's profit in a perfectly competitive market is zero in equilibrium, the wage rate for the man is given by

$$w_t^m = (1-\delta)\left(1+\ell_t^{fY}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(1+\ell_t^{fY}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}} + b. \tag{25}$$

Furthermore, Eqs. (24) and (25) yield

$$w_t^m - w_t^f = b. \tag{26}$$

From Eq. (26), we find that the male wage rate exceeds the female wage rate by the amount b in equilibrium. Therefore, the parameter b can be interpreted as representing the gender wage gap between men and women.

2.4 Childcare Services Sector

Next, we explain the profit-maximizing behavior of firms in the childcare services sector. When L_t^{fX} units of women's labor are used to produce X_t units of childcare services, the production function for external childcare services is formulated as follows:

$$X_t = \mu L_t^{fX}, \tag{27}$$

where $\mu > 1$ is the labor productivity for a firm in the childcare services sector. We treat μ as a parameter. Similar to Yakita (2018), we consider childcare services to be club goods. Therefore, provided overcrowding does not occur, many children can share childcare and facilities as a non-rivalry good. In addition, we assume that one childcare worker can take care of multiple children. Moreover, to maintain an appropriate level of childcare services for each child without overcrowding, equipment and facilities appropriate for external childcare services are also required. Specifically, to operate the childcare sector, in addition to personnel costs, the maintenance of childcare equipment and facilities, such as daycare centers and kindergartens, requires costs of $A > 0$ per unit of childcare service provision. For simplicity, we assume that such maintenance costs increase proportionally with an increase in the production of childcare services.

When the profit of the childcare sector is expressed as π_t^X, we obtain

$$\begin{aligned} \pi_t^X &= P_t X_t - w_t^f L_t^{fX} - A X_t \\ &= P_t \mu L_t^{fX} - w_t^f L_t^{fX} - A\mu L_t^{fX}. \end{aligned} \tag{28}$$

From Eq. (28), the following relationship holds in the subjective equilibrium of firms in the external childcare sector:

$$P_t = \frac{w_t^f + A\mu}{\mu}. \tag{29}$$

Considering Eq. (29), when $P_t \gtreqless w_t^f$, $\frac{\mu A}{\mu - 1} \gtreqless w_t^f$ holds; that is, if the women's wage rate is sufficiently lower than the childcare service costs per unit ($\frac{\mu A}{\mu-1}$), then the price of external childcare services (P_t) is higher than the women's wage rate. Conversely, when the wage rate for women is sufficiently higher than the cost of external childcare services per unit, the price of external childcare services is lower than the wage rate for women, and there is no demand for external childcare services. Therefore, external childcare services are not produced in cases (ii) and (iii).

3 Relationship between Capital per Couple and the Fertility Rate

In this section, we derive the equations for the relationship between capital per couple and the fertility rate in general equilibrium for cases (i), (ii), and (iii). In capital market equilibrium, the following equation holds:

$$K_{t+1} = s_t N_t. \tag{30}$$

Note that the equilibrium condition for the capital market in Eq. (30) holds in all cases (i), (ii), and (iii). Moreover, for equilibrium in the women's labor market, the relationships of $L_t^f \equiv L_t^{fY} + L_t^{fX} = N_t$ in case (i), $L_t^f = (1-h_t)N_t = L_t^{fY}$ in case (ii), and $L_t^f = 0$ in case (iii) hold.

As in Yakita's (2018) model, the economy in our model develops in the order of case (iii), case (ii), and case (i). In the following discussion, we refer to case (iii) as the early stage, case (ii) as the middle stage, and case (i) as the long-term stage. Recall that, in the early stage of economic development, the fertility rate does not depend on capital per couple and is a constant value, as seen in Eq. (21). Therefore, we focus our analysis on the middle and long-term stages of economic development.

Regarding the relationship between capital per couple and the fertility rate, we first consider the long-term stage of economic development in case (i). In this case, $X_t = (n_t N_t)/\phi$ holds in market equilibrium for childcare services. The left-hand side of this equation represents the amount of supply for childcare services, whereas the right-hand side represents the amount of demand for childcare services. In addition, $1+\ell_t^{fY} = 2 - n_t/(\mu\phi)$ can be obtained from Eq. (27). In this case, w_t^f in Eq. (24) and w_t^m in Eq. (25) can be rewritten as follows:

$$w_t^f = (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}, \tag{31}$$

$$w_t^m = (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}} + b. \tag{32}$$

From Eqs. (29), (31), and (32), the following equation can be derived:

$$\begin{aligned} n_t &= \left(\frac{\varepsilon\phi}{1+\varepsilon+\rho\theta}\right) \\ &\times \left\{ \frac{2(1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}} + b}{\frac{(1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta-1}\left[\delta\,(k_t)^{-\eta} + (1-\delta)\left(2-\frac{n_t}{\mu\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}}{\mu} + A} \right\}. \end{aligned} \tag{33}$$

Note that Eq. (33) implies that the fertility rate (n_t) is a function of the capital per couple (k_t) in the long-term stage of economic development (i.e., case (i)).

Next, we consider the middle stage of economic development (i.e., case (ii)). Recall that, in this case, the wife's time spent raising children at home is expressed as $h_t = n_t/\phi$. Thus, the following relationship holds:

$$1+\ell_t^{fY} = 2 - \frac{n_t}{\phi}. \tag{34}$$

In addition, considering Eq. (34), Eqs. (24) and (25) can be rewritten, respectively, as follows:

$$w_t^f = (1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta-1}\left[\delta\left(k_t\right)^{-\eta}+(1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}, \tag{35}$$

$$w_t^m = (1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta-1}\left[\delta\left(k_t\right)^{-\eta}+(1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}+b. \tag{36}$$

Furthermore, Eqs. (13), (35), and (36) imply that

$$\begin{aligned} n_t &= \left(\frac{\varepsilon\phi}{1+\varepsilon+\rho\theta}\right) \\ &\quad\times\left\{2+\frac{b}{(1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta-1}\left[\delta\left(k_t\right)^{-\eta}+(1-\delta)\left(2-\frac{n_t}{\phi}\right)^{-\eta}\right]^{-\frac{1+\eta}{\eta}}}\right\}. \end{aligned} \tag{37}$$

Note that Eq. (37) represents the relationship between capital per couple (k_t) and the fertility rate (n_t) in the middle stage of economic development, that is, the phase of case (ii).

4 Numerical Comparative Statics

We recapitulate our discussion so far. In our model, the degree of progress of economic development is measured by the degree of accumulation of capital per couple. Naturally, it can be interpreted that the greater the capital per couple, the more advanced the economic development. The macro economy in a country develops in the order of the phase of case (iii), the phase of case (ii), and the phase of case (i) in our model (hereafter, these are abbreviated as Phases (iii), (ii), and (i), respectively); that is, the process of economic development is segmented into three stages: the early stage, middle stage, and long-term stage. Recall that the fertility rate in general equilibrium (hereafter, referred to simply as the fertility rate) becomes a constant in the early stage of economic development. Specifically, the fertility rate equals the reciprocal of the time required to rear a pair of children, a boy and a girl, ϕ. It follows that the fertility rate during the early stage is independent of capital per couple, Thus, in our analysis in the following, we focus on Phases (ii) and (i). Figure 1 represents a conceptual diagram of the process of economic development through the three stages in our model.

In Figure 1, the horizontal axis represents the capital stock per couple and the vertical axis represents the fertility rate. In particular, note that, in Phase (i) of the graph, that is, the long-term stage, the fertility rebound is depicted.

In this section, we perform comparative statics from a quantitative perspective; that is, we numerically examine the effects of changes in life expectancy at birth, the elasticity

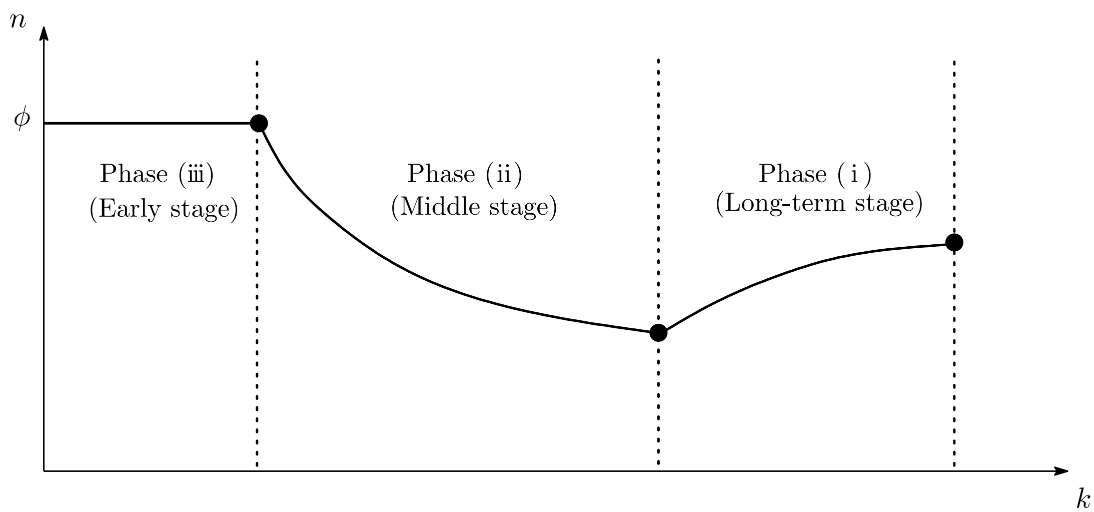

Figure 1: Relationship between capital per couple and the fertility rate through the three stages of economic development

of substitution, and the gender wage gap on the fertility rate. Prior to performing comparative statics, we look at actual data on "life expectancy at birth" and "the ratio of the population aged 65 and over to the total population" in Group of Seven (G7) member countries.[4]

Figure 2 shows changes in life expectancy at birth from 1960 to 2022 for G7 member countries

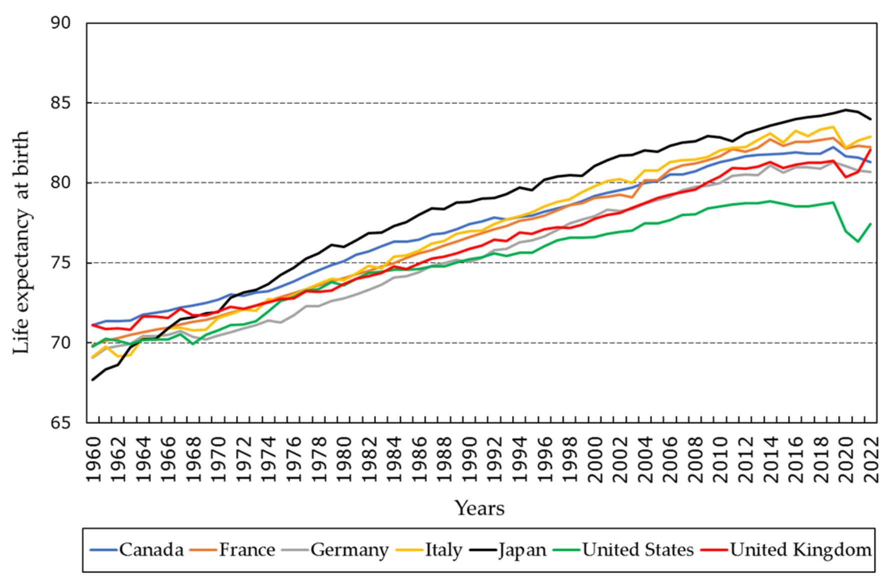

Figure 2: Life expectancy at birth for G7 member countries, 1960–2022

Table 1 shows the average life expectancy at birth in each decade from the 1960s to the 2010s for G7 member countries.

[4]These data of G7 member countries are available from the World Bank's World Development Indicators. For the details, see `https://databank.worldbank.org/source/world-development-indicators`.

Table 1: Average ife expectancy at birth for G7 member countries, 1960s–2010s

	1960s	1970s	1980s	1990s	2000s	2010s
Canada	71.8	73.6	76.2	78.1	80.1	81.8
France	70.7	72.7	75.2	77.7	80.1	82.4
Germany	70.1	71.5	74.0	76.4	78.9	80.7
Italy	70.1	72.7	75.4	78.1	80.8	82.8
Japan	70.1	74.1	77.5	79.7	82.0	83.6
United States	70.2	72.2	74.5	75.8	77.5	78.7
United Kingdom	71.4	72.6	74.7	76.7	78.8	81.1

The change in the average life expectancy at birth for each country in Table 1 indicates that the relevant value has increased for all countries from the 1960s to the 2010s. Figure 2, together with Table 1, shows the existence of broad upward trends in life expectancy at birth for all G7 member countries within the time period of the data. Japan had the highest level of life expectancy at birth among G7 member countries. It should be noted that, in Figure 2, since 2020, life expectancy at birth for G7 member countries has experienced a slight decline, and its degree varies by country. If we infer the background of such behavior, the most likely explanation is the influence of the recent coronavirus disease 2019 (COVID-19) that occurred in Wuhan City, Hubei Province, China in December 2019 and spread to almost all countries in the world in March 2020; that is, a temporary decline in life expectancy at birth in G7 member countries after 2020 may be attributable to COVID-19.

Figure 3 depicts the change in the percentage of the population aged 65 and over to the total population from 1960 to 2022 for G7 member countries.

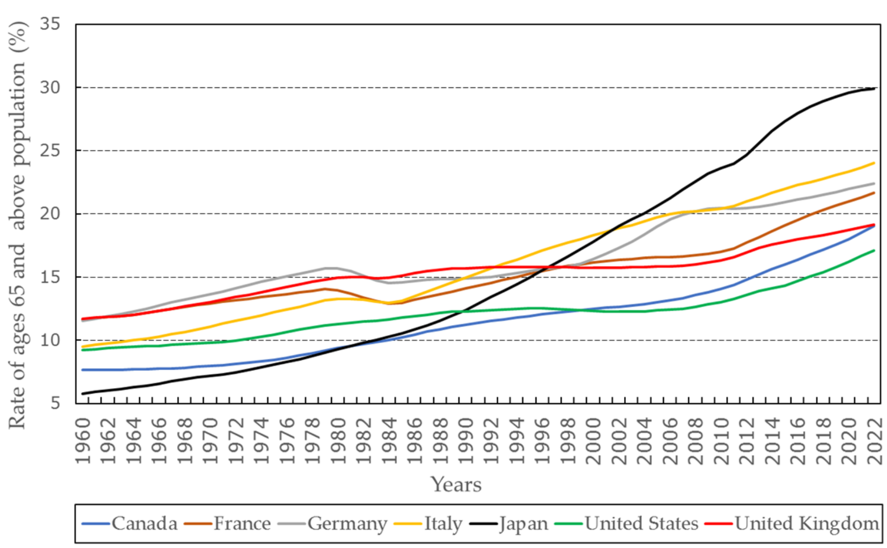

Figure 3: Ratio of the population aged 65 and over to the total population (%) for G7 member countries, 1960–2022

From Figure 3, at first glance, the rates of the population of older adults in all G7 member countries have been increasing consistently. In particular, the level of Japan's population aging rate is significantly higher than those of the other G7 member countries. The World Health Organization and United Nations define an aging society as one in which the percentage of the total population aged 65 or older exceeds 7%, an aged society as one in which this percentage exceeds 14%, and a super-aged society as one in which this percentage exceeds 21% (see Tahara, 2016). In 2022, the year of the most recent data available, among G7 countries, France, Germany, Italy, and Japan were regarded as super-aged societies because the ratio of the population aged 65 and over to the total population exceeded 21%. It should be noted that, as of 2006, Japan had already exceeded 21%, making it the fastest country among G7 countries to become a super-aged society. In addition, the number of years required to reach an aged society from an aging society is called the doubling years, and Japan's doubling years were only 25 years (see Komine and Kabe, 2009).

Table 2 shows the average percentage of the population aged 65 and over to the total population in each decade from the 1960s to the 2010s for G7 member countries.

Table 2: Average percentage of the population aged 65 and over to the total population for G7 member countries, 1960s–2010s

	1960s	1970s	1980s	1990s	2000s	2010s
Canada	7.74	8.49	10.20	11.80	13.00	15.80
France	12.17	13.48	13.42	15.11	16.51	18.83
Germany	12.44	14.70	14.94	15.41	18.59	20.93
Italy	10.13	12.11	13.51	16.54	19.47	21.77
Japan	6.40	8.02	10.50	14.75	20.44	26.63
United States	9.51	10.42	11.75	12.44	12.43	14.32
United Kingdom	12.18	13.92	15.20	15.78	15.86	17.55

From the averages from the 1960s to the 2010s shown in Table 2, an upward trend occurred in the level of population aging for each G7 member country.

From the above discussion of the data, we confirmed the evidence that the level of life expectancy at birth is steadily progressing in each G7 member country through an investigation of data on life expectancy at birth. Based on such evidence, we examine the effect of changes in life expectancy at birth (i.e., degree of longevity) on fertility at each stage of economic development. We set the values of the parameters in Eqs. (33) and (37). Specifically, the parameters that are maintained at the baseline settings are as follows: cost per unit of childcare service provision (A), gender wage gap (b), distribution parameter (δ), weight of having children on the utility of a couple (ε), substitution parameter (η), labor productivity of the firm in the childcare services sector (μ), subjective discount factor (ρ), and reciprocal of the time required to rear a pair of children, a boy and a girl (ϕ). The baseline specification is given by $A = 1.0$, $b = 1.0$, $\delta = 0.5$, $\varepsilon = 0.5$, $\eta = 2.0$, $\mu = 2.0$, $\rho = 0.7$, and $\phi = 5.0$. Regarding the parameter of life expectancy at birth (θ), we consider the values of 0.1, 0.3, 0.5, 0.7, and 0.9.

Figures 4 and 5 depict the relationship between capital per couple and the fertility rate under the parameter variations of θ in Phases (ii) and (i), respectively.

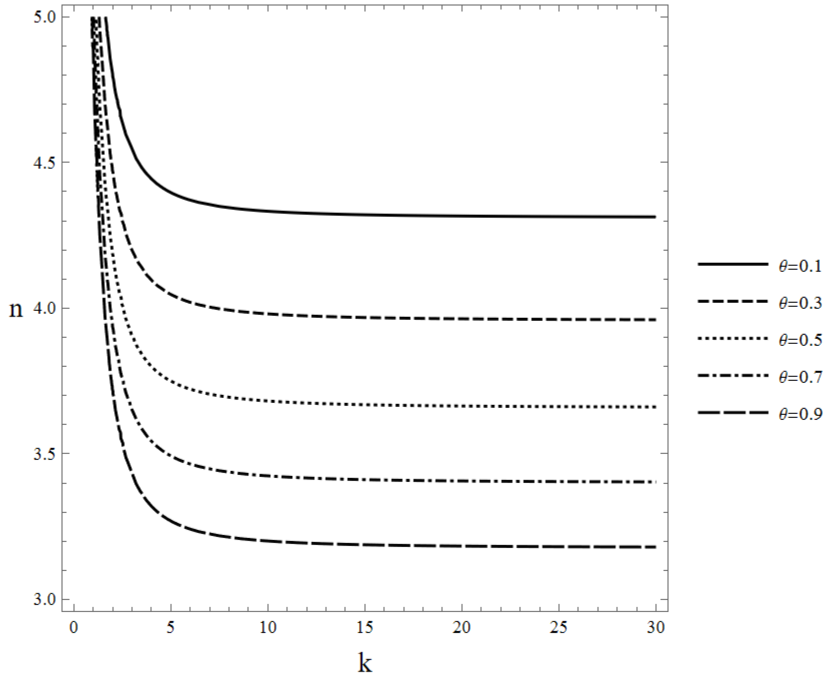

Figure 4: Influence of life expectancy at birth on the fertility rate: middle stage

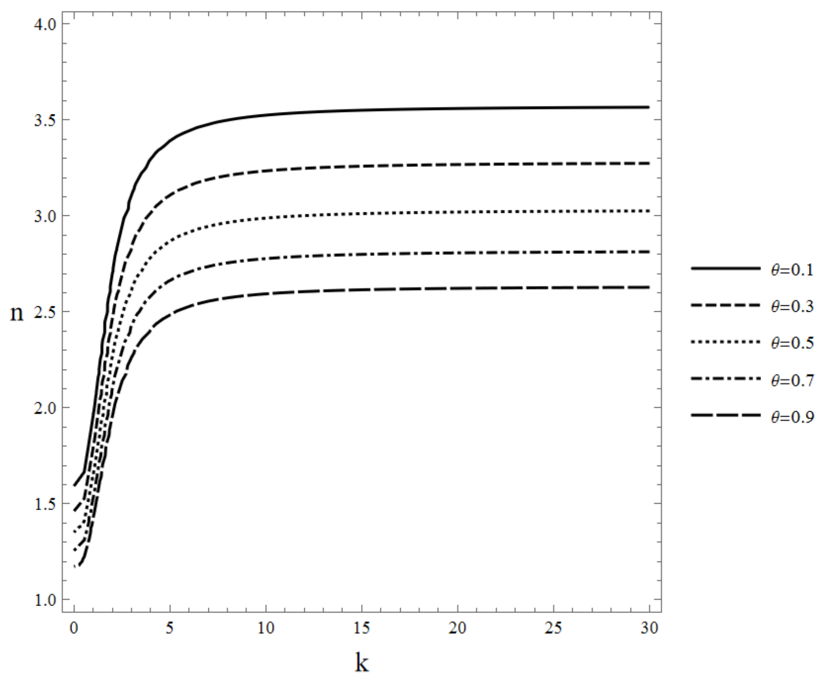

Figure 5: Influence of life expectancy at birth on the fertility rate: long-term stage

The graph in Figure 4 shows that the relationship between capital per couple and the fertility rate shifts downward when θ increases from 0.1 to 0.9. Thus, given the capital per couple, the longer that life expectancy at birth increases, the lower the fertility rate.

Moreover, under a given capital per couple, an improvement in life expectancy at birth (i.e., increase in longevity) implies an increase in the rate of the population of older adults (i.e., the share of people in the old retired period in the total population). Therefore, our model suggests that the progress of population aging leads to a decrease in the fertility rate.

Recall that Phases (ii) and (i) are the middle and long-term stages of economic development, respectively. For convenience, we consider economies in the middle stage as developing countries and consider those in the long-term stage as developed countries. Note that, generally, although population aging in developing countries does not draw as much attention as it does in developed countries, population aging is also steadily progressing in developing countries (see, e.g., Shetty, 2012; Pillay and Maharaj, 2013; Bloom and Luca, 2016). Thus, based on evidence of steady population aging for developing countries in the middle stage of economic development, which is Phase (ii), our model predicts a future fertility decline in such developing countries.

The above results are essentially the same those for the case in Figure 5, where the economy is in the long-term stage of Phase (i). For example, for a given capital per couple, an improvement in life expectancy at birth implies an increase in the rate of the population of older adults. Clearly, this implication is consistent with the trends in data on life expectancy at birth and the rate of the population of older adults for G7 member countries, as seen in the comparison of Figures 2 and 3 (or Tables 1 and 2). To summarize, with respect to Figure 5, we believe that our model successfully describes the real scenario.

Next, we investigate the influences of the elasticity of substitution on the fertility rate. Table 3 shows the values of the elasticity of substitution (σ) correlating to values indicated by the substitution parameter (η).

Table 3: Relationship between the substitution parameter and the elasticity of substitution

Substitution parameter (η)	Elasticity of substitution (σ)
0.50	0.6667
1.00	0.5000
1.50	0.4000
2.00	0.3333
2.50	0.2857

As shown in Table 3, we considered the cases of 0.50, 1.00, 1.50, 2.00, and 2.50 as values of the substitution parameter (η). The values of the elasticity of substitution between capital and labor (σ) correlating to these substitution parameter variations are 0.0677, 0.5000, 0.4000, 0.3333, and 0.2857, respectively. Regarding the average estimated values of the elasticity of substitution, Mallick (2012, Table 2) indicated that the value for 90 countries and regions (including developed and developing economies) is 0.338, the value for 14 East Asian economies is 0.737, the value for sub-Saharan African countries is 0.275, the value for 24 selected OECD member countries is 0.340, and the value for 20 selected OECD member countries (excluding East Asian economies) is 0.299. These average estimated values for the elasticity of substitution correspond to values of the substitution parameter of approximately 1.96, 0.36, 2.64, 1.94, and 2.34, respectively. Thus, the values of the substitution parameter set in Table 3 can be considered reasonable.

In the quantitative considerations for the relationship between the elasticity of substitution and the fertility rate, the substitution parameter is changed to the value indicated, whereas all other parameters remain at their baseline settings as follows: $A = 1.0$, $b = 1.0$, $\delta = 0.5$, $\varepsilon = 0.5$, $\theta = 0.9$, $\mu = 2.0$, $\rho = 0.7$, and $\phi = 5.0$. Figures 6 and 7 are graphs for the relationship between capital per couple and the fertility rate associated with different values of the substitution parameter when the economy is in the middle and long-term stages of economic development, respectively.

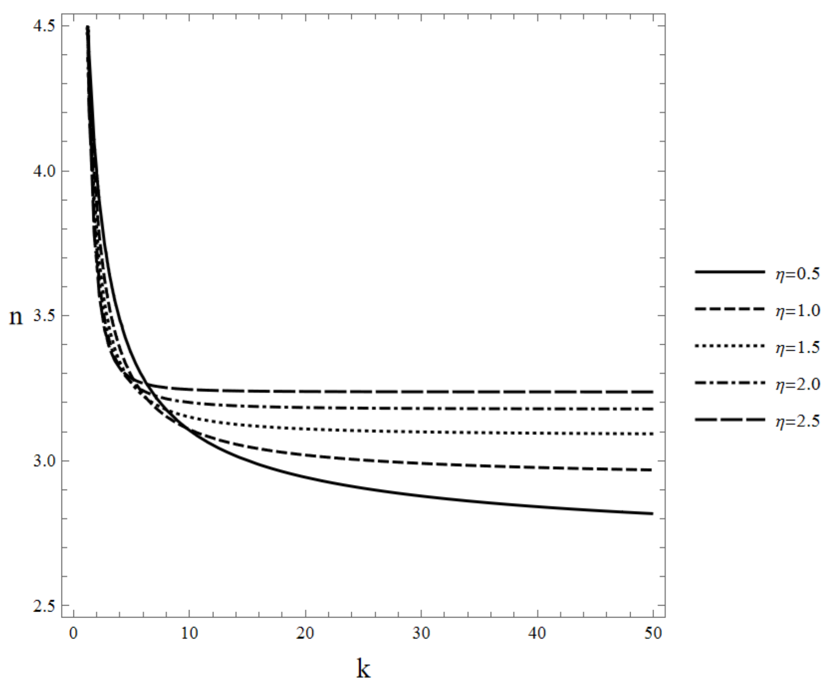

Figure 6: Influence of the elasticity of substitution on the fertility rate: middle stage

Note that Figures 6 and 7 correspond to Eqs. (37) and (33), respectively. As shown in Table 3, for positive values of the substitution parameter, the greater (smaller) the value of the substitution parameter, the smaller (greater) the elasticity of substitution.

First, we examine a scenario in which the economy is in the middle stage of Figure 6. Considering the shift in the graph corresponding to the change in the value of the substitution parameter (i.e., an increase in η from 0.5 to 2.5), the graph shifts upward as the substitution parameter takes on larger values; that is, the graph shifts upward as the elasticity of substitution (σ) decreases because, when the substitution parameter is positive, a larger value of the substitution parameter means a small value of the elasticity of substitution between capital and labor. Now we consider the values of the elasticity of substitution presented in Table 3. Within most of the values of capital per couple given, the fertility rate corresponding to a large value of the elasticity of substitution (e.g., $\sigma = 0.6667$) is smaller than the fertility rate corresponding to a large value of the

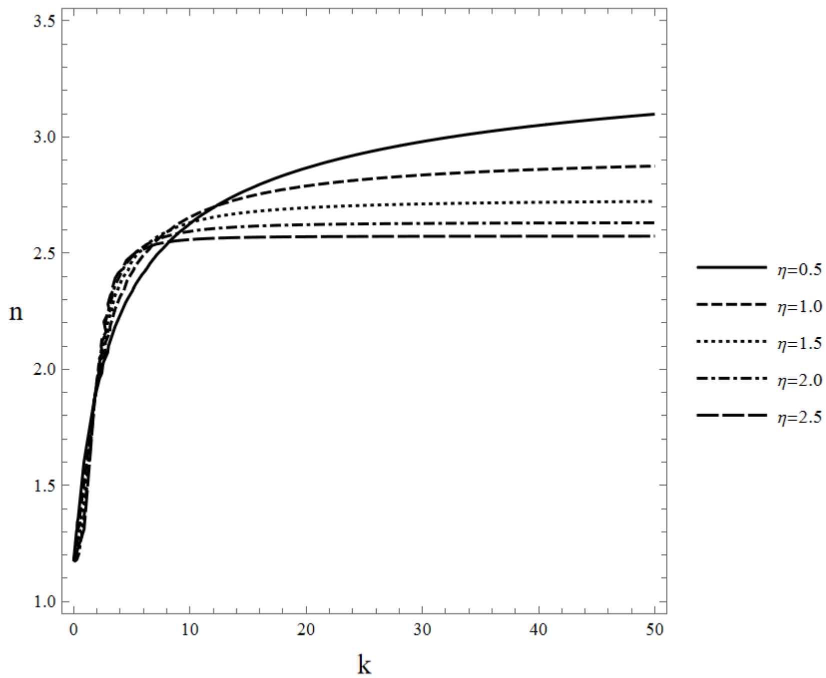

Figure 7: Influence of the elasticity of substitution on the fertility rate: long-term stage

elasticity of substitution (e.g., $\sigma = 0.2857$). This implies that, under most values of capital per couple given, an increase in the elasticity of substitution results in a decrease in the fertility rate in the scenario in which the economy is in the middle stage of economic development, that is, Phase (ii).

Next, we consider the case of the long-term stage of Figure 7. In this case, the graph shifts downward as the value of the alternative parameter (η) increases (i.e., the value of the elasticity of substitution σ decreases). Thus, given most values of capital per couple, the fertility rate corresponding to a large value of the elasticity of substitution (e.g., $\sigma = 0.6667$) is larger than that corresponding to a small value of the elasticity of substitution (e.g., $\sigma = 0.2857$). This is opposite to the result of the case in Figure 6. Specifically, when the economy is in the long-term stage of economic development, which denotes Phase (i), within most ranges of a given value of capital per couple, an increase in the elasticity of substitution between capital and labor leads to an increase in the fertility rate.

Finally, we analyze the relationship between the gender wage gap and the fertility rate. We set the values of the gender wage gap's parameter (b) to 0.1, 0.2, 0.3, 0.4, and 0.5. All other parameters remain at their baseline settings. Specifically, we set these parameters to the following values: $A = 1.0$, $\delta = 0.5$, $\varepsilon = 0.5$, $\eta = 2.0$, $\theta = 0.9$, $\mu = 2.0$, $\rho = 0.7$, and $\phi = 5.0$. Figures 8 and 9 show the relationship between capital per couple and the fertility rate under the parameter variations of b, considering the scenario in which the economy is in the middle and long-term stages of economic development, respectively.

Figure 8 shows that, as the gender wage gap narrows, the graph shifts downward. Thus, for a given capital per couple, the smaller the gender wage gap, the lower the fertility rate.

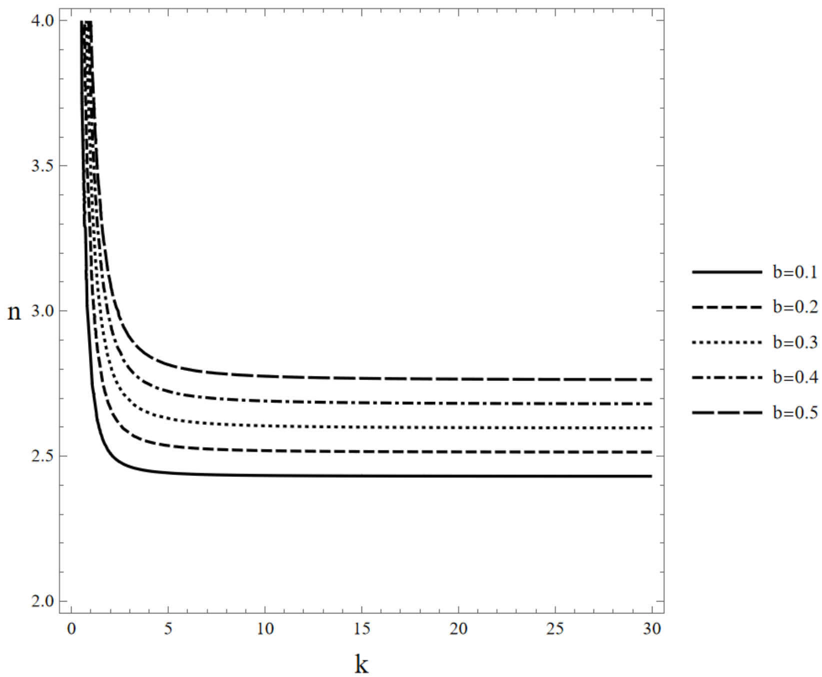

Figure 8: Influence of the gender wage gap on the fertility rate: middle stage

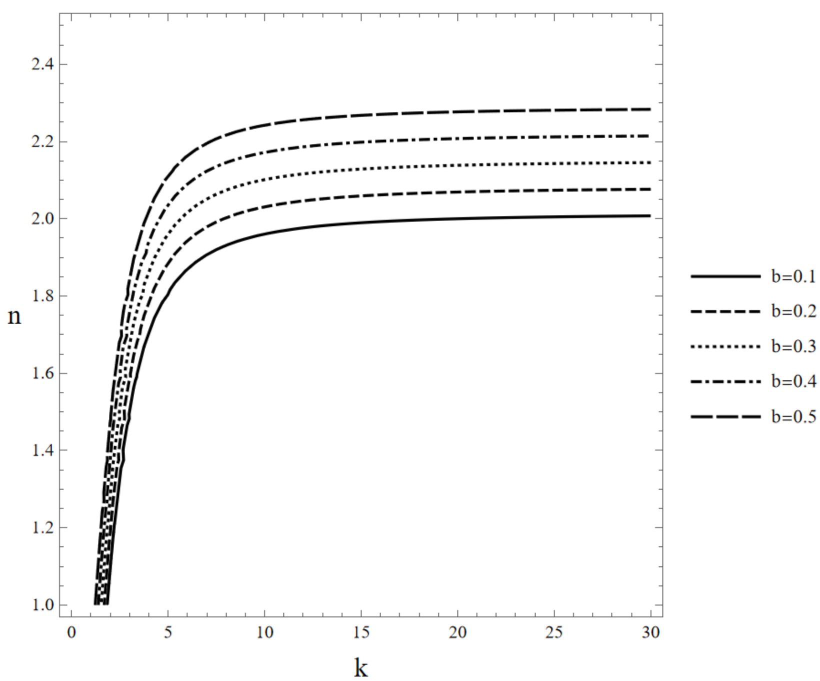

Figure 9: Influence of the gender wage gap on the fertility rate: long-term stage

As a result, when the economy is in the middle stage, which denotes Phase (ii), a narrowing of the gender wage gap leads to a decline in the fertility rate. This implication of our model is essentially the same as that for the case in Figure 9. Specifically, even when the economy is in the long-term stage, that is, Phase (i), for a given capital per couple, a narrowing of the gender wage gap results in a decline in the fertility rate.

5 Policy Implications

In this section, we discuss the policy implications of the model based on the results of the analysis in Section 4. In particular, we focus on policy issues related to increasing the fertility rate. As mentioned in Section 4, countries with economies in the middle stage, or Phase (ii), can be considered developing countries, whereas those with economies in the long-term stage, or Phase (i), can be considered developed countries.

Considering the results of the analysis in Figures 4 and 5, in both cases, for a given capital per couple, the fertility rate declined as life expectancy at birth improved (i.e., as steady progress of population aging). This is a phenomenon that is common to both developing and developed countries, and in particular, further aging of the population in the future is considered inevitable. Therefore, with other aspects constant, we can expect a decline in fertility rates in both developing and developed countries in the future. For economies in the middle stage, that is, developing countries, increasing capital per couple is a necessary policy to increase the fertility rate. This is because, as shown in Figure 1, the fertility rebound occurs through the transition from the middle stage to the long-term stage; that is, the key issue for developing countries is the policy to promote economic growth. Moreover, for developed countries that are already in the long-term stage and experiencing a fertility rebound, what measures are available to increase fertility? Figure 5 shows that once capital per couple exceeds a certain level, capital accumulation thereafter has little effect on fertility. Life expectancy at birth is expected to increase in the future. Given this outlook, developed countries with capital per couple above a certain level must consider approaches that aim to achieve both an increase in the fertility rate and sustained economic growth. For example, using an endogenous growth model, Palivos and Scotese (1996) demonstrated that socially optimal rates of fertility and economic growth can be achieved through a policy mix of public childcare support services financed by a lump-sum tax and a voucher system. The examination of the preconditions for the feasibility of such a policy mix is an important issue.

Next, we discuss policy implications based on the results of the analysis of Figures 6 and 7. In our model, when the economy is in the middle stage, an increase in the elasticity of substitution implies a decrease in the fertility rate. However, when the economy is in the long-term stage, an increase in the elasticity of substitution implies an increase in the fertility rate. Thus, simply put, in the case of developing countries, increasing fertility requires the implementation of policies that reduce the elasticity of substitution between capital and labor. Conversely, in the case of developed countries, increasing fertility requires the implementation of policies that increase the elasticity of substitution between capital and labor. We attempt to interpret the implications of our model based on the findings of Nakamura (2009), who used the concept of "mechanization." In Nakamura's (2009) model, mechanization means that the range of capital utilization expands in relation to labor. Moreover, Nakamura (2009) found the relationship between mechanization and the elasticity of substitution as follows: the greater (smaller) the elasticity of substitution between capital and labor, the easier (more difficult) mechanization becomes. Therefore, to interpret the implications of our model from the mechanization perspective of Nakamura (2009), in the case of developing countries that are in the middle stage, an increase in the fertility rate requires a decrease in the elasticity of substitution. This implies that mechanization is difficult. By contrast, in the case of developed countries that are in the long-term stage, an increase in the fertility rate requires an increase in the elasticity of substitution. This means that mechanization is easy. With the development of information and communication technology, the promotion of mechanization (i.e., the expansion of the range of capital utilization in the sense that capital is used more extensively than labor) is an inevitable trend to drive the economic activities of firms, regardless of whether they are in developing or developed countries; that is, the existence of a mechanization bottleneck may lead to a stagnation of macroeconomic performance. Thus, developed countries can achieve both higher fertility rates and mechanization, whereas developing countries face the dilemma of not being able to achieve both. The reason that some developing countries have not been able to take off on the path of economic development may be due to the difficulty of mechanization because of the low elasticity of substitution.

Finally, we examine the policy implications of the gender wage gap with respect to Figures 8 and 9. We obtained similar results for the gender wage gap for the middle and long-term stages. Specifically, the results show that a decrease in the gender wage gap leads to a decrease in fertility both in developing countries in the middle stage of the economy and developed countries in the long-term stage of economic development; that is, an increase in fertility requires an increase in the gender wage gap. In reality, however, most people would not agree to such a policy. Our model suggests that it is very difficult to achieve both a narrowing of the gender wage gap and an increase in the fertility rate simultaneously. Further study is needed on policy responses to this issue.

Incidentally, since the SDGs were adopted by the United Nations General Assembly in September 2015, interdisciplinary research on the SDGs has been actively conducted. However, it should be noted that the majority of earlier studies on the SDGs have been examined in terms of empirical approaches. By contrast, very few cases exist of analysis from the viewpoints of theoretical approaches. As one of few theoretical studies, our study is particularly aligned with the content of target 8.1 of goal 8 ("Decent Work and Economic Growth") of the SDGs. Clearly, this SDG target is closely related to economic development in a broad sense (economic growth in a narrow sense). However, to the best of our knowledge, few studies exist in which researchers developed the argument in the context of models of economic development or economic growth (e.g., one of the exceptions is Noda and Kano, 2021) regarding the issues of SDGs. Therefore, the analytical framework based on our OLG model provides part of a theoretical foundation to improve the understanding and achievement of this SDG target.

6 Conclusion

In this article, we examined the relationship between capital per couple and the fertility rate through stages of economic development (i.e., early stage, middle stage, and long-term stage) using an OLG model with an uncertain lifetime, which is an extension of Yakita's (2018) model. The intuitive explanation of how the fertility rebound occurs in our model is as follows: From the early to middle stages of economic development, the income levels of households are not very high because women are unable to secure sufficient working hours. Therefore, they cannot afford to use out-of-home childcare services operated by private firms and the fertility rate gradually declines. However, as the economy develops to the long-term stage, the environment in which women can secure sufficient working hours begins to improve and women's income levels increase. Such an increase in women's income allows them to afford to use out-of-home childcare services. This makes it possible to completely delegate childcare to external childcare services. As a result, there is an incentive to have more children. An increase in fertility during the long-term stage of economic development is precisely the phenomenon of the fertility rebound. Such a mechanism by which the fertility rebound occurs may appear similar to the models of Day (2018) and Yakita (2018). However, unlike Day (2018) and Yakita (2018), we explained the mechanism by which the fertility rebound occurs within the context of the OLG model with an uncertain lifetime. Specifically, we incorporated factors that are not considered in the models of Day (2018) and Yakita (2018), such as the survival probability in old age and the elasticity of substitution between capital and labor, into our model and derived the relational expression between capital per couple and the fertility rate.

The most important contribution of our study is that we have elucidated the relationship between the three key factors (i.e., life expectancy at birth, the elasticity of substitution, and the gender wage gap) and the fertility rate at each stage of economic development. Thus, we have obtained useful findings that have been overlooked to date and developed the research field of economics of the household. Moreover, many studies have been conducted in which researchers have empirically analyzed the fertility rebound, whereas relatively few studies have been conducted based on theoretical analysis. In particular, there are few theoretical findings from a long-term perspective based on the framework of models of macroeconomic dynamics; that is, when considering studies from both theoretical and empirical perspectives, the scenario lacks balance and a gap exists between the two. This study partially fills the aforementioned gap. Thus, our attempt to fill such a gap can be regarded as another contribution. Furthermore, as mentioned in Section 1, the discussion in this study is relevant to goal 8 (target 8.1) of the SDGs. Our contribution in this regard is to present part of a theoretical basis for this target through the construction of an analytical framework based on an OLG model with an uncertain lifetime. Therefore, the findings of this study provide useful suggestions not only for academic researchers, such as macroeconomists, population economists, or development economists, but also for governments' public policymakers because the policy implications based on our model regarding the issue of how to achieve an increase in the fertility rate provide valuable suggestions for both developing and developed countries.

The major results of this study can be summarized as follows: In our model, the fertility rate becomes a constant in the early stage of economic development. Thus, our analytical results are focused on the middle and long-term stages of economic development. First, we mention the influences of life expectancy at birth on the fertility rate. Regarding the middle stage of economic development, the longer that life expectancy at birth increases, the lower the fertility rate. Moreover, an improvement in life expectancy at birth implies an increase in the rate of the population of older adults. Therefore, our model suggests that the progress of population aging results in a decrease in the fertility rate. The above results are essentially the same for the case in which the economy is in the long-term stage. Second, we state the effects of the elasticity of substitution between capital and labor on the fertility rate. Specifically, the fertility rate corresponding to a large value of the elasticity of substitution is larger than that corresponding to a small value of the elasticity of substitution in the middle stage of economic development. By contrast, when the economy is in the long-term stage, an increase in the elasticity of substitution implies an increase in the fertility rate. Third, we describe the impacts of the gender wage gap on the fertility rate. Focusing on the middle stage of economic development, the smaller the gender wage gap, the lower the fertility rate. As a result, when the economy is in the middle stage, a narrowing of the gender wage gap leads to a decline in the fertility rate. This implication of our model is essentially the same in the long-term stage of economic development; that is, even when the economy is in the long-term stage, a narrowing of the gender wage gap results in a decline in the fertility rate.

In addition, policy implications are derived from our model analysis as follows: In our model, for convenience, we can regard economies in the middle stage of economic development as developing countries and those in the long-term stage of economic development as developed countries. We begin with the policy implications of population aging. Further aging of society is expected to be inevitable in both developing and developed countries. For economies in the middle stage, that is, developing countries, increasing capital per couple is a necessary policy to increase fertility. This is because the fertility rebound occurs as a result of the transition from the middle stage to the long-run stage; that is, a key issue for developing countries is policies that promote economic growth. In addition, for developed countries that have already entered the long-term stage, subsequent capital accumulation has little effect on fertility. Given this outlook that life expectancy at birth is expected to increase in the future, the time has come for developed countries with per-couple capital above a certain level to identify an approach that aims to both improve fertility rates and achieve sustained economic growth. With respect to the elasticity of substitution, when the economy is in the middle stage, an increase in the elasticity of substitution implies a decrease in the fertility rate. However, when the economy is in the long-term stage, an increase in the elasticity of substitution implies an increase in fertility. Thus, in simple terms, in the case of developing countries, increasing fertility requires the implementation of policies that reduce the elasticity of substitution between capital and labor. Conversely, in the case of developed countries, increasing fertility requires the implementation of policies that increase the elasticity of substitution. In this regard, interpreting the implications of this model in terms of Nakamura's (2009) mechanization concept, for developing countries in the intermediate stage, an increase in fertility requires a decrease in the elasticity of substitution. This implies that mechanization is difficult. By contrast, for developing countries in the long-run stage, an increase in fertility requires an increase in the elasticity of substitution. This implies that mechanization is easy. With the development of information and communication technology, the promotion of mechanization is an inevitable trend to drive the economic activities of firms, regardless of whether they are in developing or developed countries; that is, the existence of a mechanization bottleneck may result in a downturn of macroeconomic performance. Although developed countries can achieve both higher fertility rates and mechanization, developing countries face the dilemma of not being able to achieve both. As for the policy implications of the gender wage gap, a decrease in the gender wage gap results in a decrease in fertility both in developing countries in the middle stage of the economy and in developed countries in the long-term stage of economic development. Thus, a necessary condition for an increase in the fertility rate is a widening of the gender wage gap. In reality, government implementation of such a policy would be unacceptable to almost everyone. Our model suggests that it is very difficult to simultaneously achieve a narrowing of the gender wage gap and an increase in fertility. Further research is needed on policy responses to this problem.

Finally, we mention several research directions for the expansion of our study. In our model setting, men do not participate in childcare, only women are used in the childcare sector. In recent years, men's participation in childcare has been encouraged worldwide. Moreover, arguments that assume that men and women engage in different jobs may not be appropriate for realistic analysis. Furthermore, as indicated by Noda (2020), many countries have emphasized work–life balance in recent years. Therefore, it would be interesting to extend our model by considering the problem of the labor–leisure choice, as in the models of Barro and Sala-i-Martin (2004, Ch. 9) and Noda (2021). These points will be the subject of future research.

Acknowledgments

This work was supported in part by a Grant-in-Aid for Scientific Research (C) (20K01639) from the Japan Society for the Promotion of Science.

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