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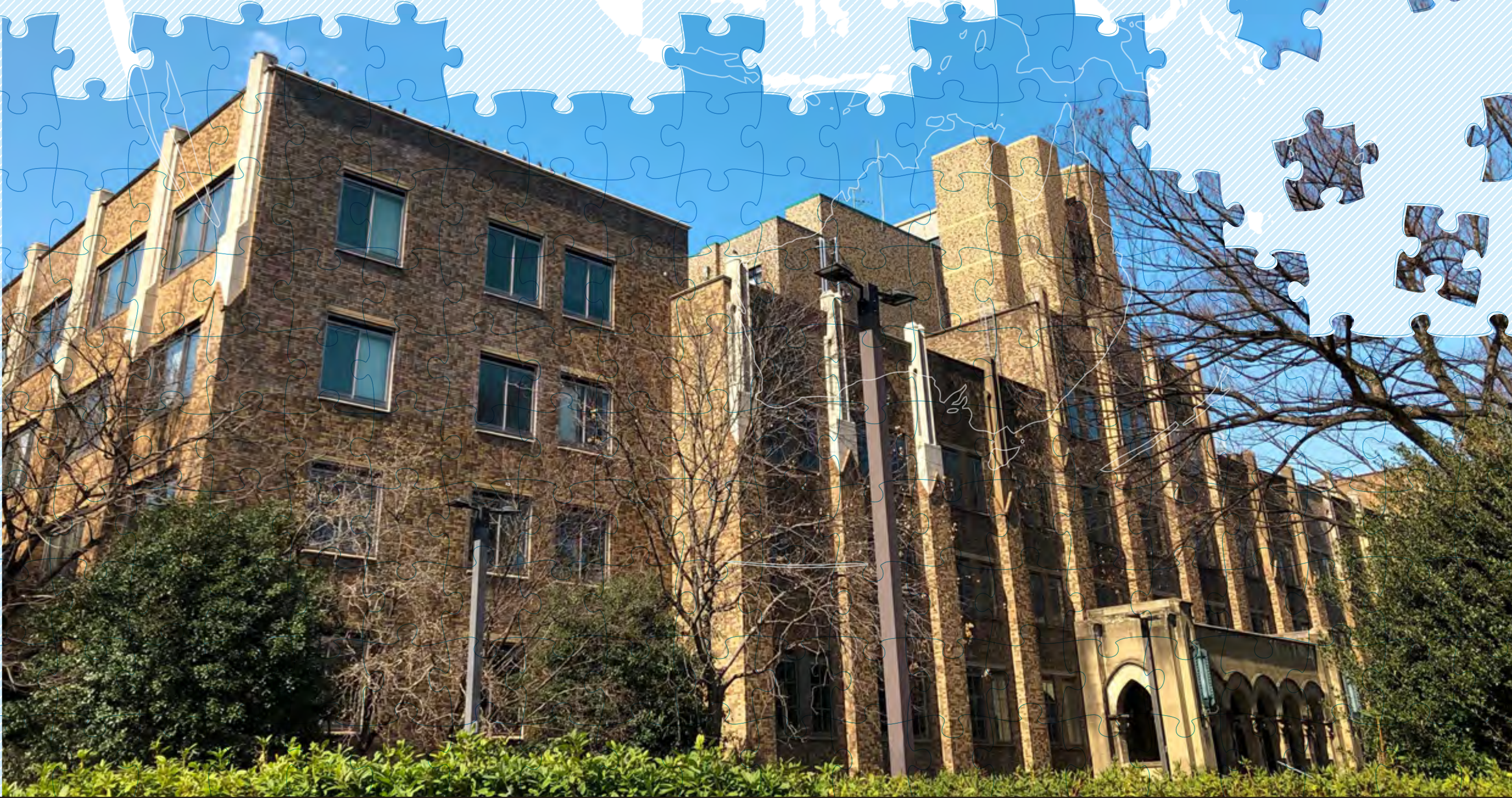
Exploring Socially Optimal Rates of Economic Growth and Fertility



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Exploring Socially Optimal Rates of Economic Growth and Fertility

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Abstract

The effectiveness of vouchers as an instrument of economic policy by local and central governments has attracted great attention in many countries. In this article, we examine the policy mix issue of the relationship between voucher system and three types of taxation methods (i.e., a consumption tax, a labor income tax, and a lump-sum tax) using an economic growth model with endogenous fertility. Our aim is to verify that the case of use of lump-sum tax is the only effective means for achieving socially optimal rates of gross domestic product (GDP) per capita and fertility when considering the policy mix combines a voucher system and one of the three taxation methods mentioned above. Moreover, we demonstrate that when labor income tax or consumption tax is adopted as a source of funding for the provision of public services, the growth rate of GDP per capita that can be achieved in a decentralized economy will be lower than the socially optimal rate, while the fertility rate in a decentralized economy will be higher than the socially optimal rate. Furthermore, we indicate that in both the model for the labor income tax case and the model for the consumption tax case, the gap between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy increases (in absolute value) as the degree of preference of an individual for having children increases.

Keywords: Economic growth, Fertility, Sustainable development goals, Taxation methods, Voucher

JEL Classification: H20, H50, J13, J18, O41

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1 Introduction

The role of vouchers has attracted the attention of economists and public policy makers because of the use of these policy tools by various governments. For example, Lubienski and Malin (2021) examined the use of voucher programs in the educational sector in the United States, where school voucher programs have been justified primarily on the basis of improving student achievement outcomes. However, as pointed out by Lubienski and Malin (2021), a series of recent empirical studies have revealed sizable negative impacts on students enrolled in voucher programs. Another recent, well-known example of the use of vouchers as a policy tool occurred in the context of the coronavirus disease 2019 (COVID-19) pandemic when governments in many countries distributed vouchers for vaccinations against COVID-19. Specifically, in Japan, the local governments of each city in Japan's prefectures distributed COVID-19 vaccination vouchers to people in advance as part of their public services (see, e.g., Shimada and Kobayashi, 2023).

The use of voucher programs can be found in many other situations in international society. However, in many cases, voucher programs have been implemented as policy tools by governments without a sufficient examination of their effectiveness from an economic perspective. Our awareness of this problem has led us to consider a rigorous theoretical analysis of the effects of voucher programs on microeconomic and macroeconomic performance to be highly important, and has prompted this study. In particular, we are interested in the effects of the policy mix of vouchers and taxation on fertility rate and long-term macroeconomic performance, because a steady decrease in fertility rate is a social concern in many developed countries. To address this issue, it is appropriate to conduct our analysis in a framework based on an economic growth model with endogenous fertility. Specifically, the purpose of this study is to theoretically examine the policy mix of voucher programs and taxation methods to finance public service provision, and clarify what type of taxation and voucher combination can achieve the socially optimal rates of fertility and economic growth.

In an earlier study, Palivos and Scotese (1996) provided an interesting discussion of the use of vouchers from the perspective of a public policy. Specifically, Palivos and Scotese (1996) considered a lump-sum tax as a source of funding for public service provision and found that socially optimal rates of economic growth per capita and fertility can be achieved by combining a voucher system and a lump-sum tax. They adopted an endogenous population growth model in the continuous-time formulation in which infinitely lived representative individuals had to choose the number of children; that is, each individual faced a fertility choice problem. Other relevant theoretical studies include Cardak (2005), Hartman and Kwon (2005), and Kim et al. (2021). Using a two-period overlapping generations model, Carsak (2005) considered the impact of education vouchers on economic growth and the evolution of income inequality. The main implications of Carsak's (2005) model are that introducing education vouchers can increase economic growth but that in general, vouchers increase income inequality. Hartman and Kwon (2005) extended the Uzawa–Lucas endogenous growth model (Lucas, 1988; Uzawa, 1965) to analyze a pollution problem and examine the possibility of pollution control. They treated pollution as an externality and showed that the social optimum can be achieved with a simple pollution tax or a voucher system. That is, either a pollution tax or a voucher system can implement a Pareto efficient solution. Based on a two-period overlapping generations model, Kim et al. (2021) analyzed deceased organ allocation policies that assign priority on organ waiting lists to voucher holders to promote deceased donor registration. They found that extending the donor priority benefits to future generations can improve the aggregate donation rate and social welfare. In particular, they determined that giving higher priority to voucher inheritors who register for donation is always beneficial, regardless of the levels of population growth and care for future generations.

Of the above-mentioned studies, our work is most closely related to Palivos and Scotese (1996). However, our study differs from theirs in that we re-examine the implications of their model when we focus on cases where labor income and consumption taxes are used as sources of funding for the provision of public services. Specifically, we verify that when comparing the three taxation methods—a lump-sum tax, a labor income tax, and a consumption tax—only the case of the lump-sum tax is an effective means of achieving socially optimal rates of gross domestic product (GDP) per capita and fertility when the policy mix combines a voucher program and taxation. Thus, we address the problem of the effectiveness of a policy combining a voucher system for health-care and education services with various taxation methods as an instrument of public policy to achieve socially optimal rates of GDP growth per capita and fertility. That is, our economic growth model with endogenous fertility is a variation of that of Palivos and Scotese (1996). Moreover, we demonstrate that when a labor income tax or a consumption tax is adopted to fund the provision of public services, the growth rate of GDP per capita that can be achieved in a decentralized economy will be lower than the socially optimal rate, whereas the fertility rate will be higher than the socially optimal rate. Furthermore, we shows that in both the model for the labor income tax case and the model for the consumption tax case, the gap between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy increases (in absolute value) as the degree of preference of an individual for having children increases.

Since the sustainable development goals (SDGs) were adopted by the United Nations General Assembly in September 2015, interdisciplinary research on the SDGs has been actively conducted. However, the majority of studies on the SDGs have adopted empirical approaches. There have been very few analyses involving theoretical approaches (a notable exception is Noda and Kano, 2021), and our study represents one of the few theoretical studies. It is particularly aligned with the content of goal 8 (target 8.1) in the SDGs ("Decent Work and Economic Growth"). Although this SDG target is closely related to economic growth, there have been few studies examining the goal or its achievement in the context of economic growth models. The analytical framework based on our economic growth model with endogenous fertility provides a theoretical foundation for better understanding and achieving this SDG target.

The remainder of this study is organized as follows. In Sections 2 and 3, we examine the cases where a labor income tax and a consumption tax, respectively, are used to fund the provision of public services. Finally, in Section 4, we describe the contributions of our study, summarize the main results, and mention future research topics.

2 The Model of the Labor Income Tax Case

2.1 A Decentralized Economy in the Case of the Labor Income Tax

Suppose that there is a closed economy composed of family-size households at time t, denoted by $N(t)$, and the government. We begin by explaining the behavior of the households in the case where a labor income tax is used to fund the provision of public services. Let $n(t) \geq 0$ be the fertility rate (i.e., the population growth rate). The size of the family changes continuously in accordance with $\dot{N}(t) = n(t)N(t)$ at each point in time, where a dot over a variable, for example $\dot{N}(t)$, denotes differentiation with respect to time.

For simplicity, we follow Barro (1990), Agénor (2008), Chen (2006), and Noda (2019, 2022) by considering a representative household/producer, which is homogeneous in terms of preferences and labor ability. The representative individuals decide on the time paths of the family's per capita consumption, $c(t)$, and the number of children, $n(t)$, to maximize the family's discounted present value of their stream of instantaneous utility (i.e., lifetime utility) over an infinite time horizon as follows:

$$U = \int_0^\infty \frac{[c(t)^\beta n(t)^{1-\beta}]^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \tag{1}$$

where parameter $\rho > 0$ is the rate of subjective time preference, and parameter $\sigma > 0$ is a parameter associated with the elasticity of intertemporal substitution between consumption at two points in time. Additionally, $\beta \in (0, 1)$ is a parameter that captures preference in terms of consumption compared to having children, while $1 - \beta$ represents the degree of preference of an individual for having children compared to consumption of goods.

We define capital used in the production of goods in the broad sense of the term as consisting of both physical and human capital (hereafter, we refer simply to capital). In this economy, there is a single homogeneous good, and capital is the only input as a factor of production when the good is produced. In addition, the homogeneous goods are regarded as the numéraire and their prices are normalized to unity.

When aggregate capital at the macroeconomic level is expressed as $K(t)$, the aggregate production function takes the following form

$$Y(t) = AK(t), \tag{2}$$

where $Y(t)$ is the flow of output and parameter A is the level of technology (i.e., the productivity of capital). Under this model setting, $Y(t)$ can also be interpreted as GDP. If we divide both sides of Eq. (2) by $N(t)$, then we obtain

$$y(t) = Ak(t). \tag{3}$$

In Eq. (3), $y(t) \equiv Y(t)/N(t)$ and $k(t) \equiv K(t)/N(t)$.

Now, we explain the household budget constraint. In the following discussion, we express the fertility rate in the decentralized economy by $n_d(t)$. Then, the budget constraint can be represented as

$$\dot{k}(t) = (1 - \tau_\ell)Ak(t) - c(t) - k(t)n_d(t), \tag{4}$$

where τ_ℓ represents the labor income tax rate which is constant over time. In our model, we treat the various types of tax rates as parameters, similar to Barro (1990), Chang et al. (2006), and Noda (2019). The dynamic optimization problem of the representative household/producer is to choose the path of the control variables $c(t)$ and $n_d(t)$ to maximize U in Eq. (1) over an infinite time horizon, subject to the budget constraint in Eq. (4) and a given initial asset, $k(0)$, as well as the labor income tax rate, τ_ℓ. By solving this problem based on an optimum control theory, the first-order conditions for maximizing U in Eq. (1) with respect to the choice of the control variables and the usual transversality condition can be obtained.

Next, we describe the government's economic activity. The government provides an amount of support equal to $g(t)$ per child to assist children to acquire the skills necessary to engage in work in the future. Specifically, the government contributes to improving children's abilities by providing public services such as health care and education. Moreover, we assume that various costs are incurred when providing public services. The government runs a balanced budget financed by a labor income tax at the rate τ_ℓ. In other words, the expenditure/revenue condition is satisfied at each point in time as follows:

$$Bn_d(t)g(t)N(t) = \tau_\ell Ak(t)N(t). \tag{5}$$

In Eq. (5), the left-hand side represents expenditure and the right-hand side represents revenue. $B \in (0, 1)$ is a parameter that reflects the cost associated with providing public services. In this case, the size of the government, η, is given by:

$$\begin{aligned}\eta &= \frac{Bn_d(t)g(t)N(t)}{Y(t)} \\ &= \tau_\ell. \end{aligned} \tag{6}$$

From Eq. (6), it is evident that the size of the government is a constant over time. The result that the size of the government is equal to the funding obtained from the labor income tax is similar to the situation in Barro's (1990) model.

Now, based on the above discussion, we examine the general equilibrium in cases where a labor income tax is imposed. Let $\widehat{\gamma}$ be the growth rate of GDP per capita designated by subscripts to indicate a decentralized economy or an economy in a socially optimal situation. Specifically, in the decentralized economy, let $\widehat{\gamma}_d$ be the equilibrium growth rate of GDP per capita when a labor income tax is used to fund public service provision. In other words, under the setup in this section, $\dot{y}(t)/y(t) \equiv \widehat{\gamma}_d$. As a result, $\widehat{\gamma}_d$ is given by

$$\widehat{\gamma}_d = \frac{1}{\beta\sigma}\Big[1 - \eta A\beta - \rho\Big]. \tag{7}$$

Moreover, the equilibrium fertility rate in a decentralized economy, $\widehat{n}_d$, is derived as

$$\widehat{n}_d = \frac{1-\beta}{\beta\sigma}\Big[\rho - A\beta(1-\eta)(1-\sigma)\Big]. \tag{8}$$

2.2 Comparison of a Decentralized Economy and the Social Optimum in the Case of a Labor Income Tax

Suppose that the economy is run by a benevolent social planner who seeks to maximize the lifetime utility of representative individuals, U in Eq. (1), through optimal choices regarding the time paths of the consumption per capita and a family's fertility rate. Let $n_s(t)$ be the fertility rate chosen by social planners. The social planner is subject to the economy's resource constraint, as follows:

$$\dot{k} = Ak(t) - c(t) - n_s(t)k(t) - Bn_s(t)g(t). \tag{9}$$

Regarding the fourth term on the right-hand side of Eq. (9), note that the higher the fertility rate, the larger is the total public service expenditure, given that the government provides an amount of support per child. Here, let $\widehat{\gamma}_s$ be the growth rate of GDP per capita in the social optimum. By solving this problem, the social planner selects the socially optimal growth rate of GDP per capita, $\widehat{\gamma}_s$, as follows:

$$\widehat{\gamma}_s = \frac{1}{\beta\sigma}\Big[(\beta+\eta)A - \rho\Big]. \tag{10}$$

In addition, the socially optimal fertility rate, $\widehat{n}_s$, is determined as follows:

$$\widehat{n}_s = \frac{1-\beta}{\beta\sigma}\Big[\rho - A\beta(1-\sigma) - A\eta\frac{1-\beta(1-\sigma)}{1-\beta}\Big]. \tag{11}$$

Next, we compare the values of the equilibrium growth rate of GDP per capita and the equilibrium fertility rate derived in a decentralized economy with those chosen by the social planner. Considering Eqs. (7) and (10), we find that

$$\widehat{\gamma}_s - \widehat{\gamma}_d \equiv \Delta\widehat{\gamma} = \frac{1}{\beta\sigma}\eta A(1+\beta) > 0. \tag{12}$$

Moreover, the relationship between Eq. (8) and Eq. (11) implies that

$$\widehat{n}_s - \widehat{n}_d \equiv \Delta\widehat{n} = -\frac{A\eta}{\beta\sigma}\Big[\beta^2(\sigma-1)+1\Big] < 0. \tag{13}$$

From Eq. (12), it is clear that $\widehat{\gamma}_s > \widehat{\gamma}_d$. In other words, the equilibrium growth rate of GDP per capita that can be achieved in a decentralized economy is lower than the socially optimal growth rate of GDP per capita. Regarding Eq. (13), because $0 < \beta < 1$ by assumption, the relationship $\widehat{n}_s < \widehat{n}_d$ holds for any value of $\sigma > 0$. Therefore, the equilibrium fertility rate in a decentralized economy is higher than the socially optimal fertility rate.

Although Palivos and Scotese (1996) focused on an analysis of the lump-sum tax case, they also solved their model using a proportional labor income tax and found that the results were qualitatively the same as those generated under the lump-sum taxation scheme. The only difference was that economic growth rates were lower and fertility rates were higher under the proportional labor income tax than under the lump-sum tax because of the additional distortion generated by the non-lump-sum tax. Our analysis confirms these statements. However, they did not analyze a voucher system in the case of a labor income tax. We address this issue in the next subsection.

In the following, we consider the effects of β on $\Delta\widehat{\gamma}$ and of β on $\Delta\widehat{n}$ in the model for the labor income tax case. Recall that β denotes the degree of preference of an individual in terms of consumption compared to having children, while $1-\beta$ denotes the degree of preference of an individual for having children compared to consumption of goods. First, we examine the relationship between β and $\Delta\widehat{\gamma}$. From Eq. (12), we find that the graph of the relationship between β and $\Delta\widehat{\gamma}$ is shown in Figure 1.

Our interest is to understand how the difference between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy changes when the degree of preference of an individual for having children changes. In our model, a higher degree of preference of an individual for having children, relative to the individual's preference for consumption, implies a smaller value of β. Figure 1 shows that the smaller the value of β, which implies a higher the degree of preference of an individual for having children, the larger the gap, $\Delta\widehat{\gamma}$, between between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy.

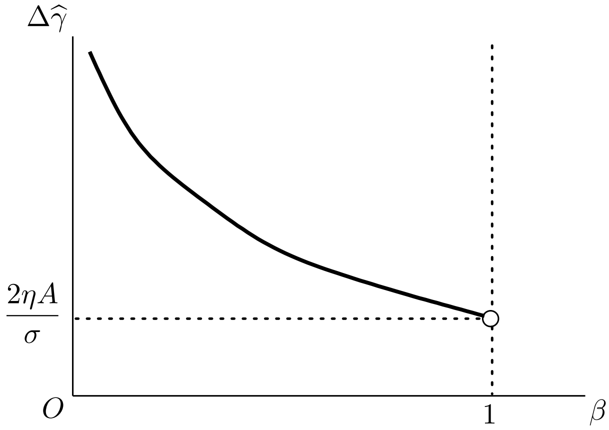

Figure 1: The relationship between β and $\Delta\widehat{\gamma}$

Next, based on Eq. (13), we analyze the effect of the degree of preference of an individual for having children on the gap between the socially optimal fertility rate and the equilibrium fertility rate in a decentralized economy. That is, we investigate the relationship between β and $\Delta\widehat{n}$. As shown in panels (a) and (b) of Figure 2, two case graphs are possible for the relationship between β and $\Delta\widehat{n}$.

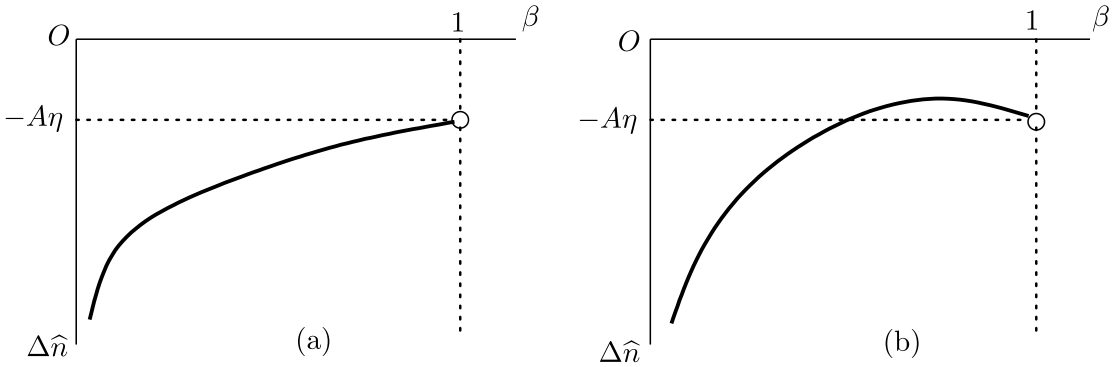

Figure 2: The relationship between β and $\Delta\widehat{n}$

Specifically, panel (a) is the graph of the case where the relation $\partial\Delta\widehat{n}/\partial\beta > 0$ holds for any value of $\beta \in (0, 1)$. On the other hand, panel (b) corresponds to the case where the relation $\partial\Delta\widehat{n}/\partial\beta > 0$ holds when the value of β is relatively small, but the relation $\partial\Delta\widehat{n}/\partial\beta < 0$ holds when β is larger than $\sqrt{1/(\sigma - 1)}$. Observing the two graphs presented in panels (a) and (b) of Figure 2 highlights the following point. Specifically, in both cases of panels (a) and (b), for a range of relatively small values of β, the smaller the value of β, the larger the gap in absolute value (i.e., $-\Delta\widehat{n}$) between the socially optimal fertility rate and the equilibrium fertility rate in a decentralized economy. This means that as the degree of preference of an individual for having children increases, the absolute value of the difference between the socially optimal fertility rate and the equilibrium fertility rate in a decentralized economy increases.

2.3 Use of the Voucher System in the Case of a Labor Income Tax

In this subsection, we consider the situation in which a voucher system is introduced by the government in a decentralized economy. Specifically, the children of the households can use the health-care and educational services provided by the government. To do so, they must exchange these public services for vouchers that the government distributes to households. However, the vouchers do not fully cover the public services that the children need; that is, the services provided in exchange for the vouchers are not sufficient to meet the children's essential needs for such services. Therefore, households must bear some of the costs of providing health-care and educational services for the children. Let $b(t)$ be the amount of essential public services per child and let $x(t)$ be the amount of public services per child that can be used in exchange for vouchers. Therefore, the cost of services per child that a household must bear equals $b(t) - x(t)$. Then, under the above settings, the budget constraint for a household with $n(t)$ children can be expressed as

$$\dot{k}(t) = (1 - \tau_\ell)Ak(t) - c(t) - k(t)n(t) - [b(t) - x(t)]n(t). \tag{14}$$

The representative household's optimization problem is to maximize U in Eq. (1) subject to the budget constraint in Eq. (14), the stock of the initial asset, $k(0)$, and the labor income tax rate, τ_ℓ. When solving the household's lifetime utility maximization problem, the household satisfies the usual first-order conditions and the transversality condition.

The size of the government η is given by

$$\begin{aligned} \eta &= \frac{b(t)n(t)N(t)}{Y(t)} \\ &= \tau_\ell. \end{aligned} \tag{15}$$

Now, we assume that the relationship $x(t) = \phi b(t)$ holds between the amount of essential public services per child and the amount actually provided through the vouchers, and that the condition $0 < \phi < 1$ is satisfied for ϕ. Let $\widehat{\gamma}_v$ be the equilibrium growth rate of GDP per capita and let $\widehat{n}_v(t)$ be the equilibrium fertility rate in a situation where a voucher system is used. Considering the relationships between the first-order conditions, Eq. (15), and $x(t) = \phi b(t)$, after some manipulation, we obtain

$$\widehat{\gamma}_v = \frac{1}{\beta\sigma}\Big[(1-\eta)A\beta + \eta A(1-\phi) - \rho\Big]. \tag{16}$$

In addition, the fertility rate under the voucher system, $\widehat{n}_v$, is determined by

$$\widehat{n}_v = \frac{1-\beta}{\beta\sigma}\Big[\rho - \beta(1-\sigma)(1-\eta)A + \eta A(1-\phi)\frac{\beta(1-\sigma)-1}{1-\beta}\Big]. \tag{17}$$

Next, we consider the value of ϕ that achieves the socially optimal values for $\widehat{\gamma}_s$ and $\widehat{n}_s$. Comparing Eqs. (10) and (16), we find a value of ϕ such that $\widehat{\gamma}_v = \widehat{\gamma}_s$, and we obtain

$$\phi = -\beta < 0. \tag{18}$$

Because $0 < \phi < 1$ and $0 < \beta < 1$, Eq. (18) clearly does not satisfy the assumptions. Therefore, the voucher system does not function effectively as a means of achieving the socially optimal growth rate of GDP per capita. Conversely, comparing equations (11) and (17), if we find a value of ϕ that satisfies $\widehat{n}_v = \widehat{n}_s$, the following relationship is derived:

$$\phi = \frac{\beta[\beta(1-\sigma) - 1]}{1-\beta} < 0. \tag{19}$$

Because we assume that $0 < \beta < 1$, the right-hand side of Eq. (19) will have a negative value. Naturally, this result does not satisfy $0 < \phi < 1$. Therefore, the voucher system is not an effective system when viewed as a policy tool for achieving a socially optimal fertility rate.

3 The Model of the Consumption Tax Case

3.1 The Decentralized Economy in the Consumption Tax Case

Here, we use the same setup as subsection 2.1, in which representative households maximize their lifetime utility, as given by Eq. (1), over an infinite time horizon. Let τ_c be the consumption tax rate. When the source of public service provision is a consumption tax, the budget constraint for the household can be expressed as

$$\dot{k}(t) = Ak(t) - (1+\tau_c)c(t) - k(t)n_d(t). \tag{20}$$

Similar to the analysis for the labor income tax case, the household's lifetime utility maximization problem is to choose the time paths of $c(t)$ and $n_d(t)$, subject to the initial asset, $k(0)$, and the consumption tax rate, τ_c, given by Eq. (20). The household's lifetime utility maximization entails the standard first-order conditions, which are obtained in the usual manner, and the transversality condition.

Assume that the government maintains a balanced budget financed by a consumption tax at rate τ_c at each point in time. The government's budget constraint for public services for children is given by

$$Bn_d(t)g(t)N(t) = \tau_c c(t)N(t). \tag{21}$$

In Eq. (21), the left-hand side denotes government spending and the right-hand side denotes government revenue. Moreover, if public services are financed by a consumption tax, the size of the government is given by

$$\begin{aligned} \eta &= \frac{Bn_d(t)g(t)N(t)}{Y(t)} \\ &= \frac{\tau_c c(t)}{Ak(t)}. \end{aligned} \tag{22}$$

The results of our model confirm that $c(t)/k(t)$ in Eq. (22) remains constant. Therefore, we find that, first, the greater the productivity of capital, the smaller is the size of government, and second, the higher the consumption tax rate, the larger is the size of government.

Considering the relationship among the first-order conditions, we can obtain the equilibrium growth rate of GDP per capita, $\widetilde{\gamma}_d$, and the equilibrium fertility rate, $\widetilde{n}_d$, in the decentralized economy in the consumption tax case. Specifically, the equilibrium growth rate of GDP per capita is expressed as

$$\widetilde{\gamma}_d = \frac{1}{\beta\sigma}(A\beta - \rho). \tag{23}$$

The equilibrium fertility rate is represented as

$$\widetilde{n}_d = \frac{1-\beta}{\beta\sigma}\Big[\rho - (1-\sigma)A\beta\Big]. \tag{24}$$

3.2 A Comparison of the Decentralized Economy and Social Optimum in the Consumption Tax Case

Now, we turn to the social planner's optimization problem. A social planner faces a resource constraint given by

$$\dot{k}(t) = Ak(t) - c(t) - n(t)k(t) - Bn(t)g(t). \tag{25}$$

Similar to the analysis of the social planner's optimization problem in subsection 2.2, the social planner seeks to maximize the lifetime utility of representative individuals, U in Eq. (1), through optimal choices regarding the time paths of the consumption per capita and a family's fertility rate, under the conditions of the resource constraint in Eq. (25) and the initial asset $k(0)$. If we solve this problem, the socially optimal growth rate of GDP per capita, $\widetilde{\gamma}_s$, and the socially optimal fertility rate, $\widetilde{n}_s$, can be obtained. As a result, the socially optimal growth rate of GDP per capita is given by

$$\widetilde{\gamma}_s = \frac{1}{\beta\sigma}\Big[(\beta+\eta)A - \rho\Big]. \tag{26}$$

Regarding the socially optimal fertility rate, we obtain

$$\widetilde{n}_s = \frac{1-\beta}{\beta\sigma}\Big[\rho - A\beta(1-\sigma) - A\eta\frac{1-\beta(1-\sigma)}{1-\beta}\Big]. \tag{27}$$

To consider the growth rate of GDP per capita and the fertility rate, we compare the values found in a decentralized economy with those in a socially optimal situation. First, from Eqs. (23) and (26), the comparison of growth rates of GDP per capita in these situations is as follows:

$$\widetilde{\gamma}_s - \widetilde{\gamma}_d \equiv \Delta\widetilde{\gamma} = \frac{A\eta}{\beta\sigma} > 0. \tag{28}$$

Next, the comparison of fertility rates from Eqs. (24) and (27) is as follows:

$$\widetilde{n}_s - \widetilde{n}_d \equiv \Delta\widetilde{n} = -\frac{A\eta}{\beta\sigma}\Big[1 - \beta(1-\sigma)\Big] < 0. \tag{29}$$

As can be easily confirmed, Eqs. (28) and (29) lead to the same results as those in the case where a labor income tax is used to fund public service provision. Specifically, Eq. (28) implies that $\widetilde{\gamma}_s > \widetilde{\gamma}_d$. That is, the equilibrium growth rate of GDP per capita that can be achieved in a decentralized economy is lower than the socially optimal growth rate of GDP per capita. Moreover, when we recall that $0 < \beta < 1$ by assumption, we find from Eq. (29) that $\widetilde{n}_s < \widetilde{n}_d$ holds for any value of $\sigma > 0$. Therefore, the equilibrium fertility rate in a decentralized economy is higher than the socially optimal fertility rate.

We now explore the effects of β on $\Delta\widetilde{\gamma}$ and of β on $\Delta\widetilde{n}$ in the model for the consumption tax case. First, we examine the relationship between β and $\Delta\widehat{\gamma}$. Figure 3 depicts the relationship between β and $\Delta\widehat{\gamma}$, based on Eq. (28).

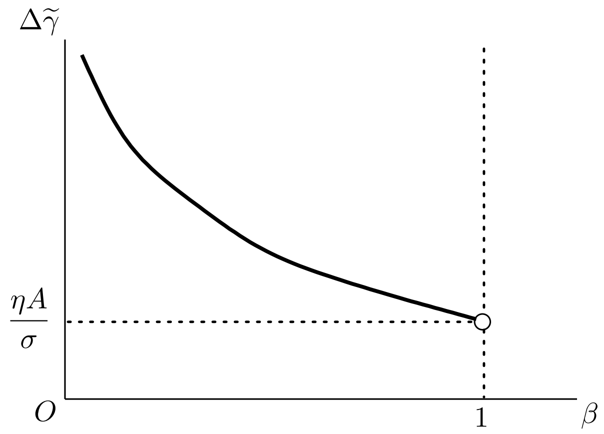

Figure 3: The relationship between β and $\Delta\widetilde{\gamma}$

As can be seen from Figure 3, a negative relationship exists between β and $\Delta\widehat{\gamma}$. This means that as the degree of preference of an individual for having children increases, the difference between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy increases.

Next, we consider the effect of β on $\Delta\widetilde{n}$. From Eq. (29), the relationship between β and $\Delta\widetilde{n}$ can be depicted as in Figure 4.

As shown in Figure 4, a positive relationship exists between β and $\Delta\widetilde{n}$. When interpreting this result, it should be noted that $\Delta\widetilde{n}$ varies over a range of negative values, corresponding to changes in the value of β, as is the case in Figure 2. Specifically, the positive relationship between β and $\Delta\widetilde{n}$ as indicated in Figure 4 implies that the absolute value of the difference between the socially optimal fertility rate and the equilibrium fertility rate in a decentralized economy increases as the degree of preference of an individual for having children increases.

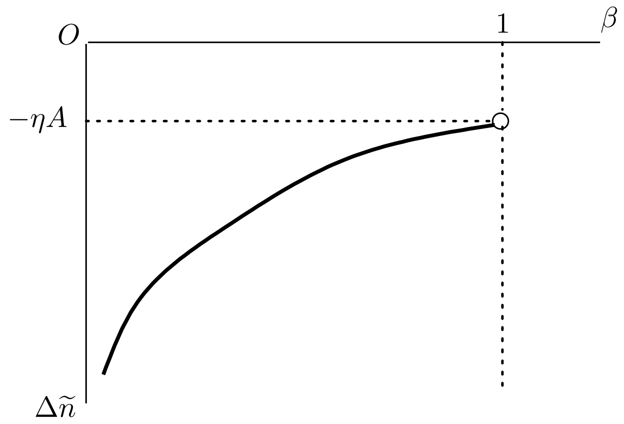

Figure 4: The relationship between β and $\Delta\widetilde{n}$

3.3 Use of the Voucher System in the Consumption Tax Case

Similar to the analysis in subsection 2.3, we first derive the equilibrium growth rate of GDP per capita and the equilibrium fertility rate when a voucher system is introduced by the government. Regarding the equilibrium growth rate of GDP per capita, considering the relationship among the the first-order conditions, Eq. (22), and $x(t) = \phi b(t)$ yields

$$\widetilde{\gamma}_v = \frac{1}{\beta\sigma}\Big[A\beta - \rho + \eta A(1-\phi)\Big]. \tag{30}$$

Regarding the fertility rate, we obtain

$$\widetilde{n}_v = \frac{1-\beta}{\beta\sigma}\Big[\rho - A\beta(1-\sigma) - \eta A(1-\phi)\frac{1-\beta(1-\sigma)}{1-\beta}\Big]. \tag{31}$$

To consider the value of ϕ under which the socially optimal growth rate of GDP per capita is achieved, we compare Eqs. (26) and (30), which yields

$$\phi = 0. \tag{32}$$

Then, to obtain the value of ϕ required to achieve a socially optimal fertility rate, if we compare Eqs. (27) and (31), ϕ must be set as

$$\phi = 0. \tag{33}$$

Because $0 < \phi < 1$ by assumption, Eqs. (32) and (33) clearly do not satisfy this assumption. Therefore, the voucher system does not function effectively as a means of achieving the socially optimal rates of GDP growth per capita and fertility.

4 Conclusion

In this study, we compared the analytical results of a voucher system in the case of the lump-sum tax presented by Palivos and Scotese (1996) with cases where labor income and consumption taxes are used to fund the provision of public services in the context of an economic growth model with endogenous fertility. In addition, we analyzed which of these three methods of taxation would achieve the socially optimal rates of GDP growth per capita and fertility through a policy mix involving a voucher system and taxation.

Vouchers are now widely used by local and central governments to achieve various public policy goals in both developed and developing countries. Moreover, vouchers are often used as a component of a policy package along with other tools, such as the policy mix that we discussed in this study, which involves a voucher system and taxation methods. Therefore, we consider that examining the use of voucher programs as a policy instrument has become increasingly important. However, to the best of our knowledge, although many studies have empirically analyzed the relationship between voucher programs and macroeconomic performance, relatively few theoretical analyses have been conducted. In particular, there is a lack of accumulated theoretical findings based on the framework of a long-term perspective using economic growth models. In other words, when examining the theoretical and empirical approaches to research on voucher systems, there is an imbalance between such approaches and a large gap that should be addressed by additional theoretical studies.

This study partially fills such gap by constructing the model to analyze the relationship between growth rate of GDP per capita and fertility rate in cases where labor income and consumption taxes are used as sources of funding for the provision of public services. Specifically, we contribute to the development of the theoretical research on issue of policy mix regarding achievement of socially optimal rates of GDP growth per capita and fertility by combining a voucher program and taxation. Our economic growth model with endogenous fertility provides meaningful policy implications not only for academic researchers such as population economists and development economists, but also for public policymakers within government. Thus, the presentation of new insights for policy practitioners as well as economic researchers is the most important contribution of our study. Moreover, another contribution of our study is the demonstration that the use of lump-sum tax is the only effective means for achieving the desired policy goal when a government attempts to achieve socially optimal rates of GDP growth per capita and fertility using a policy mix involving a voucher system and taxation methods. That is, the results of our analysis partially complement and support the study of Palivos and Scotese (1996). Furthermore, the discussion in this study is relevant to Goal 8 (target 8.1) in the SDGs. In particular, our theoretical analysis of the appropriate policy to achieve a socially optimal growth rate of GDP per capita can indicate meaningful measures into the conditions necessary to enhance the standard of living of people in developing and developed countries. To the best of our knowledge, there are no studies in which researchers have discussed the context of models of economic growth regarding goal 8 (target 8.1) of the SDGs. Thus, this study can be regarded as a stepping stone in a theoretical approach to research on goal 8 (target 8.1) of the SDGs and the analytical framework based on our model provides a theoretical foundation for better understanding and achieving this SDG target. This is a significant contribution in the interdisciplinary aspect of our study from a global perspective.

The major results of our study can be summarized as follows. In the case of a labor income tax, in a decentralized economy, the equilibrium rates of GDP growth per capita and fertility that can be achieved are lower and higher, respectively, than the socially optimal rates. The voucher system does not function effectively as a means of achieving the socially optimal growth rate of GDP per capita or the socially optimal fertility rate when a labor income tax is used to fund public service provision. Moreover, we obtain similar results in the consumption tax case; that is, again, the equilibrium rates of GDP growth per capita and fertility achieved in a decentralized economy are lower and higher, respectively, than the socially optimal rates. Thus, the voucher system is not an effective system when viewed as a policy tool for achieving the socially optimal growth rate of GDP per capita and the socially optimal fertility rate in the case where a consumption tax is adopted to fund public service provision. Therefore, when comparing the cases of labor income, consumption, and lump-sum taxes as sources of funding for public service provision, the voucher system works effectively only in the case where a lump-sum tax is used as a policy instrument. That is, the socially optimal results (i.e., the socially optimal rates of GDP growth per capita and fertility) are achieved only in the situation where the policy mix implemented involves a lump-sum tax combined with a voucher system. Furthermore, we examined the effect of the degree of individual preference for having children on the gap between growth rates and the gap between fertility rates. The results of our analysis are as follows. In both the model for the labor income tax case and the model for the consumption tax case, the gap between the socially optimal growth rate of GDP per capita and the equilibrium growth rate of GDP per capita in a decentralized economy increases (in absolute value) as the degree of preference of an individual for having children increases. Similarly, the gap between the socially optimal fertility rate and the equilibrium fertility rate in a decentralized economy increases (in absolute value) as the degree of preference of an individual for having children increases in both the model for the labor income tax case and the model for the consumption tax case.

Finally, we mention several directions for expansion of our research. In this study, we defined capital in the broad sense as including both physical capital and human capital, and developed our discussion in a situation where firms have a linear production function. Thus, our model falls into the category of models known as AK models (see, e.g., Barro and Sala-i-Martin, 2004). Although the AK model is very useful for simplifying analyses, it has some limitations. For example, it cannot analyze the effects of the elasticity of substitution between physical and human capital on the fertility rate and the growth rate of GDP per capita. Thus, one direction in which our model can be extended in future is by analyzing the case of a production function with a constant elasticity of substitution that clearly distinguishes physical capital and human capital. Another important extension involves generalizing the model by introducing not only fertility rate but also mortality rate as an endogenous variable, similar to the model of Blackburn and Cipriani (1998). Furthermore, as pointed out by Noda (2020), many countries have emphasized work–life balance in recent years. Therefore, it would be interesting to derive the socially optimal level of work–life balance in conjunction with the socially optimal rates of GDP growth per capita and fertility by considering the problem of the labor–leisure choice, as in the models of Barro and Sala-i-Martin (2004, Ch.9) and Noda (2021). These potential extensions are deferred to future research.

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