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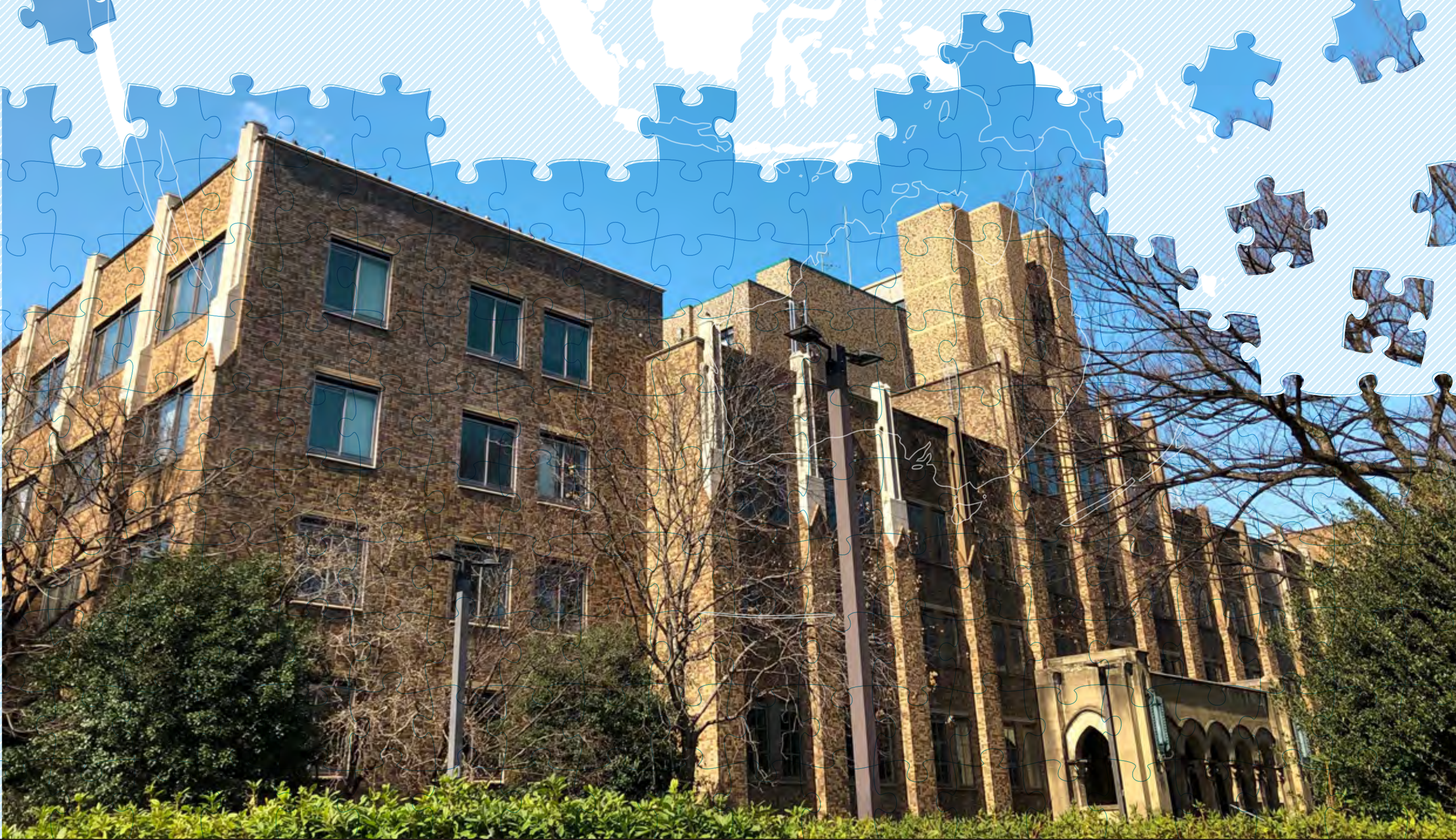
Population Aging and Taxation Methods



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Population Aging and Taxation Methods

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August 6, 2025

Abstract

The purpose of this study is to present a theoretical rationale for selecting one taxation method from the three options of labor income, consumption, and sales taxes as a source of funding for the provision of public services using a computable overlapping generations (OLG) model. We particularly focus on advanced economies that are encountering steady population aging and a prolonged downturn in economic growth, such as most Group of Seven (G7) member countries. Specifically, we numerically analyze the issue of the selection of taxation methods, considering the effects of the progress of population aging on social welfare, assuming a scenario in which the government sets a target level for the economic growth rate based on the future growth outlook for G7 member countries. Our OLG model implies that a sales tax should be selected when the purpose of the government is to maximize social welfare. However, given a low target economic growth rate of less than 1.5% per annum, higher social welfare can be achieved when a labor income tax is used rather than either a consumption tax or sales tax as a source of funding for the provision of productive public services. Moreover, a progress of the population aging in a low-growth economy decreases social welfare for all taxation methods. Furthermore, our analytical results suggest that the use of a labor income tax to fund the provision of public services is a rational choice given a state of low growth performance when the progression of population aging is unavoidable.

Keywords: Consumption tax, Labor income tax, Population aging, Sales tax, Sustainable development goals

JEL Classification: E62, H50, O40

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1 Introduction

Public services provided by the government play an important role in the production activities of the private sector. The financial resources that enable the provision of public services vary widely in terms of taxes, public bonds, and other sources. Therefore, the following questions arise: What means should be used as a source of finance to provide public services? What is the relation between steadily progressing population aging and the funding source for the provision of public services, under the scenario of normalized low growth performance that many developed countries face? Obviously, understanding what taxation method is desirable in an aged society as a means of financing public service provision is a key issue in macroeconomic policy. In particular, we are interested in advanced economies facing a steady progress of population aging and a prolonged stagnation of economic growth, such as most Group of Seven (G7) member countries. However, when we focus on low-growth economies with a progress of the population aging, insufficient findings are available regarding appropriate financial resources for providing public services.

Government spending that aids an increase in private-sector productivity and contributes to economic growth is termed productive government expenditure. One of the most influential studies arguing for the importance of productive government expenditure in achieving sustained economic growth was conducted by Barro (1990).[1] The features and implications of Barro's (1990) model are summarized as follows: The analytical framework is an endogenous growth model based on the settings of an infinite-lived representative agent in which a labor income tax is used to finance public service expenditure. The key results derived by Barro's (1990) model are that there exists an inverted U-shaped relationship between the labor income tax rate and the growth rate of gross domestic product (GDP), and the tax rate that maximizes the GDP growth rate and the tax rate that maximizes social welfare become equal within the context of the Cobb–Douglas production function. Note that because Barro's (1990) model assumes an individual as a household-producer in the economy, a labor income tax can also be regarded as a flat-rate tax imposed by the government on the revenue of each firm (hereafter, sales tax).

The seminal work of Barro (1990) was followed by an extremely large number of studies involving applied research using a wide range of extended models, adding new elements such as congestion effects, a household's utility-enhancing consumption of public services, and public capital stock that contribute to the productivity of private-sector firms (see, e.g., Irmen and Kuehnel (2009) for a survey of theoretical studies on the applications of Barro's (1990) model).

[1] Kneller *et al.* (1999) performed panel data analysis regarding 22 countries that are members of the Organisation for Economic Co-operation and Development (OECD) and confirmed that Barro's model was supported from the data of 22 OECD member countries.

In this study, we specifically focus on the importance of productive government expenditure (i.e., productive public services) as a flow variable and do not consider any financing methods other than taxation, such as public bonds and public–private partnerships. The purpose of this study is to clarify which taxation method should be selected under various conditions when three options, a labor income tax, consumption tax, and sales tax, are available as a source of funding for the provision of productive public services using a computable overlapping generations (OLG) model with an uncertain lifetime.

Similar to our study, those of Watanabe *et al.* (2015), Gómez (2016), and Pham (2019) focused on the role of productive government expenditure as an engine of economic growth and developed the argument of Barro (1990) in several directions. For instance, Watanabe *et al.* (2015) analyzed the problem of choosing between a labor income tax and a consumption tax as a source of finance for the provision of public services within the framework of an OLG model and derived the following results. When the government places importance on the utility of future generations, the use of a consumption tax results in greater social welfare than the use of a labor income tax. However, if the government does not emphasize the utility of future generations, this does not necessarily hold. Moreover, when the government sets a target annual GDP growth rate of between 2% and 4%, the use of a labor income tax generates greater social welfare than the use of a consumption tax. Gómez (2016) considered a constant elasticity of substitution (CES) production function and examined the possibility of congestion in relation to the use of public services within the context of an extended version of Barro's (1990) model. The financial resources available for the provision of public services are derived from a labor income tax and lump-sum tax. When there is congestion, the first-best solution is obtained through a particular income tax rate, and the lump-sum tax is adjusted to satisfy the government's budget constraints. However, if there is no congestion, the income tax rate is set to zero and public services are financed using the lump-sum tax. In Gómez's (2016) model with a CES production function, the elasticity of substitution between private-sector input and public-sector input plays an important role in the macroeconomic implications of the model. Specifically, when the optimal fiscal policy is implemented, the long-term GDP growth rate and the level of social welfare are high in an economy with a high elasticity of substitution. Pham (2019) introduced the externality of consumption to Barro's (1990) model and examined the policy implications of the extended model. When such an externality of consumption exists, people consider not only their own consumption but also their social status based on the ratio of their own consumption to the perceived average consumption of society. In Pham's (2019) model, a labor income tax or a lump-sum tax is adopted to finance the provision of public services. The results suggest that the level of social welfare decreases when the status-seeking behavior of each individual with the aim of "keeping up with the Joneses" promotes economic growth. When a high GDP growth rate is achieved using a lump-sum tax, a high level of social welfare might be caused by a labor income tax in the case in which the size of the government is sufficiently large.

As is well known, the steady increase in population aging is a non-negligible issue facing many countries.[2] Of the above-mentioned studies, only that conducted by Watanabe *et al.* (2015) addressed the issue of population aging. Government revenue through a labor income tax may decrease in association with the progression of population aging. Additionally, a labor income tax causes a decline in the equality of the intergenerational burden. Thus, Watanabe *et al.* (2015) argued that a consumption tax is preferable to a labor income tax as a means of financing public services. However, it should be noted that Watanabe *et al.* (2015) excluded a sales tax from their analysis. In this regard, as for a consumption tax, we consider that a sales tax is also beneficial in terms of the equality of the intergenerational burden. Therefore, we extend the OLG model of Watanabe *et al.* (2015) by considering the case in which the government imposes a sales tax on private-sector firms in addition to a labor income tax and consumption tax. Moreover, similar to Watanabe *et al.* (2015), we assume a scenario in which the government sets a target level for the GDP growth rate. Specifically, given a low target GDP growth rate, which is based on the future growth outlook for G7 member countries, we analyze the effects of the steady progress of population aging on the tax rates of the three taxation methods and the corresponding levels of social welfare. Thus, this study can be regarded as an effort to complement that of Watanabe *et al.* (2015).

The main results are summarized as follows: when the government aims to maximize social welfare, this goal is achieved in the case in which a sales tax is used. Thus, if the government seeks to achieve the objective of maximizing social welfare, it is rational to choose a sales tax as a source of financing for the provision of productive public services. However, from a different point of view from that above, under the assumption that the government sets a low target GDP growth rate of less than 1.5% per annum, a labor income tax should be chosen as the source of funding for the provision of productive public services because it provides the highest level of social welfare compared with cases in which the other taxes are adopted. Moreover, under a low target GDP growth rate of 1.18% per annum, social welfare is expected to decrease gradually with the progress of population aging, regardless of the taxation method. In such a scenario, the level of social welfare in the case in which a labor income tax is used is consistently higher than in the case in which either a consumption tax

[2] Goh *et al.* (2020) used data from Japan to explain that population aging would have a long-term effect on several key macroeconomic variables. In general, population aging tends to be perceived as a phenomenon that is limited to developed countries. However, population aging in developing countries is also an issue that needs to be monitored. For example, Teixeira *et al.* (2017) claimed that the effect of population aging on economic growth in developing countries was underestimated.

or sales tax is adopted. Thus, in a low-growth economy facing the steady progression of population aging, a labor income tax has an advantage over a consumption tax and sales tax in terms of the level of social welfare. Furthermore, under a low target GDP growth rate of 1.18% per annum, technological progress at the economy-wide level has the effect of reducing tax rates for all taxation methods.

The remainder of this article is organized as follows: In Section 2, we explain the basic structure of an extended version of the OLG model presented by Watanabe *et al.* (2015) in a closed economy consisting of households, firms, and the government. In Section 3, we derive the GDP growth rate and social welfare in cases in which a labor income tax, a consumption tax, and a sales tax are used as the source of funding for the provision of productive public services. In Section 4, we conduct numerical analysis based on an extended version of the OLG model presented by Watanabe *et al.* (2015) and examine the implications of the model under various conditions. Finally, in Section 5, we describe the contributions of our study, summarize the main results, and mention future research topics.

2 The Model

2.1 Households

We begin by describing the basic setup of the model regarding households (individuals). All individuals can live for only two periods at most. The first period is termed the young-age period, whereas the second is termed the old-age period. Moreover, all individuals engage in work only in the young-age period and inelastically provide one unit of labor. The cohort created at the beginning of period t is called generation t. For simplification, the young population of each generation is assumed to be 1. Individuals of generation $t \geq 0$ are able to live through their young-age period. However, their survival into the old-age period is uncertain. More specifically, people of generation $t \geq 0$ live in their old-age period with a probability of $p \in (0, 1)$, but die at the beginning of the old-age period with a probability of $1 - p$. In our model, p is treated as a parameter. Therefore, the total population (in terms of expected value) in each period is $1 + p$. The ratio of older people to the total population in a country is expressed as $p/(1 + p)$. Additionally, we assume the presence of risk-neutral insurance companies and a perfectly competitive private annuity market based on Yaari's (1965) study. In this case, if the interest rate in period t is expressed as r_t, the rate of return on pensions received by individuals in generation t who continue to live in their old-age period is $(1 + r_{t+1})/p$. The budget constraint of individuals in generation $t \geq 0$ in their young-age period is given by

$$(1 + \tau_c)c_{1,t} + s_t = (1 - \tau_\ell)w_t, \tag{1}$$

where $c_{1,t}$ denotes the consumption of individuals in generation t in their young-age period, s_t denotes the annuitized savings of individuals in generation t in their young-age period, τ_c denotes the consumption tax rate, τ_ℓ denotes the labor income tax rate, and w_t denotes the wage rate in period t. τ_c and τ_ℓ satisfy $0 < \tau_c$ and $0 < \tau_\ell < 1$. We also assume that individuals do not leave an estate on their death. Therefore, the budget constraint of individuals in generation t in their old-age period is given by

$$(1+\tau_c)c_{2,t+1} = \left(\frac{1+r_{t+1}}{p}\right)s_t, \tag{2}$$

where $c_{2,t+1}$ denotes the consumption of individuals in generation t in their old-age period.

The lifetime utility of individuals in generation $t \geq 0$ is expressed as u_t. This is formulated as

$$u_t = \log c_{1,t} + p\beta \log c_{2,t+1}, \tag{3}$$

where the term $\beta \in (0,1)$ is a subjective discount factor. Households consider the conditions presented in Eqs. (1) and (2) and maximize their lifetime utility in keeping with Eq. (3). When individuals in generation t displays utility-maximizing behavior, their consumption in their young-age period is given by

$$c_{1,t} = \frac{(1-\tau_\ell)w_t}{(1+p\beta)(1+\tau_c)}. \tag{4}$$

Furthermore, their consumption in their old-age period is given by

$$c_{2,t+1} = \frac{\beta(1+r_{t+1})(1-\tau_\ell)w_t}{(1+p\beta)(1+\tau_c)}. \tag{5}$$

Based on Eqs. (4) and (5), the ratio of consumption in the young-age period to that in the old-age period is expressed as $c_{2,t+1}/c_{1,t} = \beta(1+r_{t+1})$, which specifies the lifetime consumption allocation of individuals.

2.2 Firms

We assume that all firms have identical production technology. Using an approach similar to those used by Barro (1990), Alesina and Rodrik (1994), and Watanabe *et al.* (2015), the role of public services in contributing to firms' production activities is incorporated into the model.[3] Specifically, productive public services (hereafter, referred to simply as public services) are provided to the private production sector without charging user fees and without

[3] As noted by Barro (1990), these model settings depend on the findings of the pioneering empirical study by Aschauer (1989), who conducted regression analysis of data on the United States economy. Moreover, empirical research by Chu *et al.* (2020) demonstrated that the real GDP per capita growth rate would increase by 0.05 percentage points if the percentage of non-productive government expenditure was reduced by 1 percentage point and allocated to an increase in productive government expenditure in a high-income economy.

generating the congestion effect. Illustrative examples of public services include law and order services and public health services, such as the recent coronavirus disease 2019 (COVID-19) vaccination program. Given public services, firms use capital and labor to produce a single (homogeneous) good. This homogeneous good is regarded as the numéraire and its price is normalized to 1.

Let $Y_{j,t}$ be firm $j's$ production amount in period t, $K_{j,t}$ be the capital input, and $L_{j,t}$ be the labor input. When the total number of firms is represented by F, which is a sufficiently large number, the aggregates of the variables are $Y_t \equiv \sum_{j=1}^{F} Y_{j,t}$, $K_t \equiv \sum_{j=1}^{F} K_{j,t}$, and $L_t \equiv \sum_{j=1}^{F} L_{j,t}$. Notably, the relationship $L_t = 1$ holds, based on the assumption.

The production function of firm j is given by

$$Y_{j,t} = AK_{j,t}^{\theta}(G_t L_{j,t})^{1-\theta}. \tag{6}$$

In Eq. (6), G_t denotes the total amount of public services, A denotes an index of technology at the economy-wide level to which all firms have access (i.e., the baseline technology in a country), θ denotes the elasticity of output with respect to capital, and $1-\theta$ denotes the elasticity of output with respect to public services. A and θ are treated as parameters and satisfy the conditions $A > 0$ and $0 < \theta < 1$.

We assume that the government imposes a tax on the output (revenue) of firms at the rate of $\tau_f \in (0, 1)$. The depletion rate of private sector capital is assumed to be 1. Therefore, the capital rental rate equals $1 + r_t$. The after-tax profit $\pi_{j,t}$ of firm j is given by

$$\pi_{j,t} = (1-\tau_f)Y_{j,t} - w_t L_{j,t} - (1+r_t)K_{j,t}.$$

Each firm is a price-taker. When firms pursue profit maximization, the rental rate of capital becomes

$$1 + r_t = (1-\tau_f)A\theta G_t^{1-\theta} K_t^{\theta-1}. \tag{7}$$

Additionally, the wage rate becomes

$$w_t = (1-\tau_f)(1-\theta)AG_t^{1-\theta} K_t^{\theta}. \tag{8}$$

The aggregate production function in an equilibrium is given by

$$Y_t = AK_t^{\theta} G_t^{1-\theta}. \tag{9}$$

Therefore, the aggregate production function has constant returns to scale for K_t and G_t.

2.3 Government

There are three options regarding sources of finance for the provision of public services: a labor income tax, a consumption tax, or a sales tax. Government revenue is derived solely from taxation and expenditure is solely in relation to public services. We assume that the government maintains a balanced budget in each period. Considering a labor income tax, consumption tax, and sales tax simultaneously, the government's budget constraint is given by

$$G_t = \tau_\ell w_t + \tau_c c_{1,t} + \tau_c p c_{2,t} + \tau_f Y_t. \tag{10}$$

The left- and right-hand sides of Eq. (10) represent expenditure and revenue, respectively. Like Barro (1990), to simplify the analysis, we assume that government revenue through each taxation method and public services have a one-to-one relationship. In the next section, we assume various conditions in which each taxation method is used to facilitate an understanding of the different results derived using different taxation methods.

3 Economic Growth and Social Welfare

3.1 Labor income tax

First, we consider the case in which the government uses only a labor income tax as the source of finance for public service expenditure. In this case, because the relationship $\tau_f = \tau_c = 0$ holds, Eq. (10) can be rewritten as follows:

$$G_t = \tau_\ell w_t. \tag{11}$$

Substituting Eq. (11) into Eq. (8), we obtain

$$w_t = A^{\frac{1}{\theta}} (1-\theta)^{\frac{1}{\theta}} \tau_\ell^{\frac{1-\theta}{\theta}} K_t. \tag{12}$$

Moreover, substituting Eq. (11) into Eq. (7) yields

$$1 + r_t = A^{\frac{1}{\theta}} \theta (1-\theta)^{\frac{1-\theta}{\theta}} \tau_\ell^{\frac{1-\theta}{\theta}}. \tag{13}$$

From Eq. (13), r_t is constant. Furthermore, Eqs. (11) and (12) lead to

$$G_t = A^{\frac{1}{\theta}} (1-\theta)^{\frac{1}{\theta}} \tau_\ell^{\frac{1}{\theta}} K_t. \tag{14}$$

Therefore, using Eqs. (9) and (14), we obtain

$$Y_t = A^{\frac{1}{\theta}} (1-\theta)^{\frac{1-\theta}{\theta}} \tau_\ell^{\frac{1-\theta}{\theta}} K_t. \tag{15}$$

From Eq. (15), the GDP growth rate equals the capital growth rate.

In the capital market equilibrium, the relationship $K_{t+1} = s_t$ holds; that is, the total amount of savings in period t equals the capital stock at the beginning of period $t+1$. This relationship can be rewritten as follows:

$$K_{t+1} = \frac{p\beta}{1+p\beta}(1-\tau_\ell)w_t. \tag{16}$$

Let γ_ℓ be the gross growth rate of GDP in an equilibrium when a labor income tax is used as the funding source. Thus,

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{p\beta(1-\tau_\ell)A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}}\tau_\ell^{\frac{1-\theta}{\theta}}}{1+p\beta} \equiv \gamma_\ell. \tag{17}$$

Because the total population in each period is a constant value, $1+p$, the GDP growth rate equals the GDP growth rate per capita. If the labor income tax rate that maximizes γ_ℓ is expressed as $\widehat{\tau}_\ell$, the relationship $\widehat{\tau}_\ell = 1-\theta$ holds; that is, the labor income tax rate that maximizes the GDP growth rate equals the elasticity of output with respect to public services. Therefore, the labor income tax rate that maximizes the GDP growth rate is unaffected by population aging.

Next, we consider the labor income tax rate that maximizes social welfare. When a labor income tax is used, social welfare is expressed as W_ℓ. The social welfare function is given by

$$W_\ell = \frac{p\beta}{\sigma}\log c_{2,0} + \sum_{t=0}^{\infty}\sigma^t u_t, \tag{18}$$

where $\sigma \in (0,1)$ denotes the social discount factor. Consequently, Eq. (18) can be rewritten as

$$W_\ell = \frac{1+p\beta}{(1-\sigma)^2}\log\gamma_\ell + \frac{p\beta}{\sigma(1-\sigma)}\log\frac{1+r}{p} + \frac{p\beta+\sigma}{\sigma(1-\sigma)}\log K_0 - \frac{1}{1-\sigma}\log p\beta. \tag{19}$$

Let τ_ℓ^* denote the labor income tax rate that maximizes social welfare. From Eq. (19), we obtain

$$\tau_\ell^* = \frac{(p\beta+\sigma)(1-\theta)}{(1-\theta+\theta\sigma)p\beta+\sigma}. \tag{20}$$

Therefore, the relationship $\tau_\ell^* > \widehat{\tau}_\ell$ holds. Moreover, Eq. (20) implies that $\partial\tau_\ell^*/\partial p > 0$; that is, the labor income tax rate that maximizes social welfare increases in line with increased aging of the population.

3.2 Consumption tax

In the case in which the government uses only a consumption tax as the source of finance for public service expenditure, the relationship $\tau_\ell = \tau_f = 0$ holds. Thus, the government's budget constraint is given by

$$G_t = \tau_c c_{1,t} + \tau_c p c_{2,t}. \tag{21}$$

Using Eqs. (4) and (5), Eq. (21) can be rewritten as

$$G_t = \frac{\tau_c}{(1+\tau_c)(1+p\beta)}\Big[w_t + p\beta(1+r_t)w_{t-1}\Big]. \tag{22}$$

Let γ_c represent the gross growth rate of GDP in this case. For γ_c, we obtain

$$\gamma_c = \left[A(1-\theta)\Big(\frac{p\beta}{1+p\beta}\Big)^{\theta}\Big(\frac{\tau_c}{(1+\tau_c)(1+p\beta)}\Big)^{1-\theta}\Big(1+\frac{\theta(1+p\beta)}{1-\theta}\Big)^{1-\theta}\right]^{\frac{1}{\theta}}. \tag{23}$$

Note that the growth rate of GDP equals the growth rate of wages. The relationship between r_t and γ_c can be written as

$$1+r_t = \frac{\theta}{1-\theta}\frac{1+p\beta}{p\beta}\gamma_c. \tag{24}$$

From Eq. (24), r_t is a constant. Additionally, Eq. (22) implies that the growth rate of public services equals the growth rate of wages. Moreover, the capital market clearing condition is given by

$$K_{t+1} = s_t = \frac{p\beta}{1+p\beta}w_t.$$

Therefore, the growth rate of capital stock equals the growth rate of wages. As a result, the relationship $\gamma_c \equiv Y_{t+1}/Y_t = K_{t+1}/K_t = w_{t+1}/w_t$ holds in equilibrium. Note that Eq. (23) implies that $\partial\gamma_c/\partial\tau_c > 0$; that is, unlike in the case of a labor income tax, there is no finite consumption tax rate that maximizes the GDP growth rate.

Next, we consider the consumption tax rate that maximizes social welfare. Let W_c be social welfare in the case of using a consumption tax as the source of finance for public service expenditure. W_c is given by

$$\begin{aligned} W_c \;\; = \;\; & \frac{1+p\beta}{(1-\sigma)^2}\log\gamma_c + \frac{p\beta}{\sigma(1+\sigma)}\log\frac{1+r}{p} - \frac{p\beta+\sigma}{\sigma(1+\sigma)}\log(1+\tau_c) \\ & + \frac{p\beta+\sigma}{\sigma(1-\sigma)}\log K_0 - \frac{1}{1-\sigma}\log p\beta. \end{aligned} \tag{25}$$

We express the consumption tax rate that maximizes W_c as τ_c^*. From Eq. (25), τ_c^* is given by

$$\tau_c^* = \frac{1-\theta}{\theta(1-\sigma)}. \tag{26}$$

We obtain the following implications from Eq. (26). First, the higher the capital elasticity of output, the lower the consumption tax rate that maximizes social welfare. Second, the higher the social discount factor, the higher the consumption tax rate that maximizes social welfare. A high social discount factor means that the government places importance on the utility of future generations. Finally, the consumption tax rate that maximizes social welfare is not related to population aging.

3.3 Sales tax

We now consider the case in which the government uses only a sales tax as the source of finance for public service expenditure. Because the relationship $\tau_\ell = \tau_c = 0$ holds in this case, the government's budget constraint can be written as

$$\begin{aligned} G_t &= \tau_f Y_t \\ &= (\tau_f A)^{\frac{1}{\theta}} K_t. \end{aligned} \tag{27}$$

Government spending as a percentage of GDP is often regarded as an indicator of the size of the government. When the government uses only a sales tax as the source of finance for public service expenditure, the size of the government is equal to the sales tax rate.

Using Eqs. (8) and (27), we obtain

$$w_t = (1-\tau_f)(1-\theta) A^{\frac{1}{\theta}} \tau_f^{\frac{1-\theta}{\theta}} K_t. \tag{28}$$

Additionally, the relationship between Eqs. (7) and (27) leads to

$$1 + r_t = A^{\frac{1}{\theta}} \theta (\tau_f^{\frac{1-\theta}{\theta}} - \tau_f^{\frac{1}{\theta}}). \tag{29}$$

Therefore, Eq. (29) implies that r_t is a constant in equilibrium.

In our model, the relationship $K_{t+1} = s_t$ holds in equilibrium in the capital market. This can be rewritten as

$$\begin{aligned} K_{t+1} &= s_t \\ &= \frac{p\beta}{1+p\beta} w_t. \end{aligned} \tag{30}$$

Substituting Eq. (28) into Eq. (30) leads to

$$\frac{K_{t+1}}{K_t} = \frac{p\beta}{1+p\beta}(1-\tau_f)(1-\theta) A^{\frac{1}{\theta}} \tau_f^{\frac{1-\theta}{\theta}}. \tag{31}$$

Considering the relationship among Eqs. (27), (28), and (30), the growth rates of variables Y_t, K_t, G_t, and w_t become equal in equilibrium. Let γ_f be the gross growth rate of GDP when the government uses only a sales tax as the source of finance for public service expenditure. From Eq. (31), we obtain

$$\gamma_f = \frac{p\beta}{1+p\beta}(1-\theta)A^{\frac{1}{\theta}}(\tau_f^{\frac{1-\theta}{\theta}} - \tau_f^{\frac{1}{\theta}}). \tag{32}$$

When the sales tax rate that maximizes the GDP growth rate is expressed as $\widehat{\tau}_f$, the relationship $\widehat{\tau}_f = 1-\theta$ is obtained from Eq. (32). This relationship implies that maximizing the GDP growth rate requires setting the sales tax rate equal to the elasticity of output with respect to public services. Following Tsoukis and Miller (2003), Agénor (2013), and Agénor and Montiel (2015), we can interpret this relationship as "the Barro rule" with respect to the sales tax, which is regarded as an important benchmark in tax policy analysis.

Next, we derive the sales tax rate that maximizes social welfare. If social welfare in the case of using a sales tax as the source of finance for public service expenditure is represented by W_f, this is given by

$$W_f = \frac{p\beta}{\sigma(1-\sigma)}\log\frac{1+r}{p} + \frac{p\beta+\sigma}{\sigma(1-\sigma)}\log K_0 + \frac{(1+p\beta)}{(1-\sigma)^2}\log\gamma_f - \frac{1}{1-\sigma}\log p\beta. \tag{33}$$

Furthermore, differentiating both sides of Eq. (33) yields

$$\frac{dW_f}{d\tau_f} = \left[\frac{(1+p\beta)}{(1-\sigma)^2}\frac{1}{\gamma_f} + \frac{p\beta}{\sigma(1-\sigma)}\frac{1}{1+r}\frac{1+p\beta}{p\beta}\frac{\theta}{1+\theta}\right]\frac{d\gamma_f}{d\tau_f}. \tag{34}$$

From Eq. (34), we find that if public services are provided using finance derived from a sales tax, the sales tax rate that maximizes the GDP growth rate and the sales tax rate that maximizes social welfare are equal. Therefore, when the sales tax rate that maximizes social welfare is expressed as τ_f^*, the relationship $\tau_f^* = \widehat{\tau}_f = 1-\theta$ holds. In the case of using only a sales tax as the funding source, $1-\theta$ is equivalent to the size of the government.

In our model, the tax rates that maximize the GDP growth rate are the same when a labor income tax and sales tax are used as funding sources, whereas the tax rate that maximizes social welfare is higher when a labor income tax is used than when a sales tax is used; that is, the relationship $\tau_f^* = \widehat{\tau}_f = \widehat{\tau}_\ell < \tau_\ell^*$ holds.

4 Numerical Analysis

In this section, based on the OLG model presented above, we set specific parameter values and perform numerical analysis to examine the implications of the model. We performed the computations for our numerical analysis using Python.

Like Barro and Sala-i-Martin (2004, Ch.3) and Watanabe *et al.* (2015), we assume that one period in our OLG model is 30 years. The parameter values of the baseline settings for the index of the technological level (A), the subjective discount factor (β), and the capital elasticity of output (θ) are set to $A = 13.6296$, $\beta = 0.77$, and $\theta = 0.25$. Of these three parameters, we use the same values as Watanabe *et al.* (2015) for A and β. Regarding the capital elasticity of output (θ), if the markets of the factors of production are competitive, then θ is equal to the capital share (i.e., the fraction of GDP used to rent physical capital). Sturgill (2012) estimated that the average capital share for 46 countries, including both developed and developing countries, was 0.22 and the average capital share for G7 member countries was 0.25. In our quantitative analysis, we adopt the average of G7 member countries as the parameter value of the capital share (i.e., the capital elasticity of output). Watanabe *et al.* (2015) used the parameter value of 0.3 for the capital share, which is close to the estimated value of 0.299 for the capital share in Finland presented by Sturgill (2012). With regard to the parameters for the probability of survival into the old-age period (p) and the social discount factor (σ), we follow Watanabe *et al.* (2015) by setting $p = 1.000, 0.968$, and 0.8712 and $\sigma = 0.847, 0.770$, and 0.693. We also set the initial value of capital stock to $K_0 = 1$ as a baseline specification, similar to Watanabe *et al.* (2015). However, note that for the initial value of capital stock, the parameters A, p, and θ, there are cases in which we change the above baseline specification, depending on the objectives of the quantitative analysis, as described later. All other parameters remain at their baseline settings.

The tax rate that maximizes social welfare is called the optimal tax rate. Based on the parameter values presented above, the calculation of social welfare and the gross GDP growth rate when the government selects the optimal tax rate yields the results shown in Table 1.

Table 1: Social welfare and the gross GDP growth rate corresponding to the optimal tax rate

		Labor income tax			Consumption tax			Sales tax		
p	σ	τ_ℓ^*	W_ℓ	γ_ℓ	τ_c^*	W_c	γ_c	τ_f^*	W_f	γ_f
1.000	0.847	0.764	515.51	499.92	19.608	572.27	2965.91	0.750	577.49	1187.50
	0.770	0.772	241.23	498.24	13.044	247.57	2758.82	0.750	267.64	1187.50
	0.693	0.782	144.39	495.35	9.772	136.94	2570.56	0.750	158.47	1187.50
0.968	0.847	0.764	506.59	490.79	19.608	565.15	2989.78	0.750	567.77	1165.73
	0.770	0.772	236.98	489.20	13.044	244.55	2781.02	0.750	263.06	1165.73
	0.693	0.781	141.78	486.45	9.772	135.30	2591.24	0.750	155.70	1165.73
0.8712	0.847	0.763	479.42	461.52	19.608	543.36	3055.43	0.750	538.18	1095.96
	0.770	0.771	224.02	460.18	13.044	235.30	2842.08	0.750	249.14	1095.96
	0.693	0.779	133.84	457.86	9.772	130.27	2648.14	0.750	147.30	1095.96

In the following discussion, unless otherwise distinguished, the gross growth rate of GDP is simply referred to as the GDP growth rate. The results presented in Table 1 are summarized as follows: The level of social welfare is highest in the case of the optimal tax rate of sales tax. Therefore, selecting a sales tax as the source of finance for the provision of public services is rational when the government intends to maximize social welfare. This is a new and meaningful finding that cannot be found in the model setting of Watanabe *et al.* (2015). The reason that the level of social welfare is highest in the case in which a sales tax is adopted to finance the provision of public services compared with the other taxation methods is that the optimal sales tax rate has the lowest value among the three taxation methods.

When comparing the levels of social welfare corresponding to the optimal labor income tax rate and the optimal consumption tax rate, we find that the magnitude relationship differs depending on the values of the parameter; that is, it is not possible to conclude which is smaller (or larger) for each social welfare corresponding to the optimal labor income tax rate and the optimal consumption tax rate. Specifically, we focus on the case of $p = 0.968$. If $\sigma = 0.847$, then the level of social welfare related to the optimal consumption tax rate is larger than the level of social welfare related to the optimal labor income tax rate; that is, the relation $W_\ell = 506.59 < W_c = 565.15$ holds. However, if $\sigma = 0.693$, then the level of social welfare related to the optimal labor income tax rate is larger than the level of social welfare related to the optimal consumption tax rate. This implies that the relation $W_\ell = 141.78 > W_c = 135.30$ holds. Note that the smaller σ means that the government places more emphasis on the utility of the current generation than on the utility of future generations. Therefore, the above results suggest that the less the government places importance on the utility of future generations, the more social welfare in the case of the optimal labor income tax rate tends to be larger than social welfare in the case of the optimal consumption tax rate. Additionally, recall that the ratio of older people to the total population in a country is expressed as $p/(1+p)$ in our model. Thus, an increase in the probability of survival (i.e., an improvement in life expectancy), which is an increase in p, means an increase in the rate of the population of older adults (i.e., the progress of population aging). From Table 1, the level of social welfare corresponding to the optimal tax rate for all taxation methods increases in conjunction with the progress of population aging. Furthermore, we find that the following characteristics exist between the progression of population aging and the socially optimal tax rates for the three taxation methods. Regarding the labor income tax, the socially optimal tax rate increases as the population ages. However, in the case of a consumption tax and sales tax, the socially optimal tax rates for these taxation methods are unrelated to the progress of population aging.

With respect to the GDP growth rate, it is highest when a consumption tax is adopted as a source of financing for public service provision. The background of this result can be interpreted as follows: In the cases of a labor income tax and sales tax, there exists an inverse U-shaped relationship between the tax rate and the GDP growth rate. Therefore, a high tax rate does not necessarily mean a high GDP growth rate for these taxes. As confirmed in Eq. (23), however, the higher the consumption tax rate, the higher the GDP growth rate in the case in which a consumption tax is adopted. Moreover, Table 1 shows that the value of the optimal consumption tax rate is much higher than the values of the optimal tax rates for the other taxation methods. Thus, it follows that the GDP growth rate corresponding to the optimal consumption tax rate has the highest value compared with the case of the other taxation methods. Furthermore, as shown in Table 1, it should be noted that the GDP growth rate corresponding to the optimal tax rates for all taxation methods increases with the progress of population aging.

Next, we investigate how the optimal tax rate, GDP growth rate, and social welfare vary depending on the capital elasticity of output (θ) when the government selects the optimal tax rate of each taxation method. In this study, we focus on the case of $p = 0.968$ and $\sigma = 0.770$, and consider $\theta = 0.15, 0.20, \ldots, 0.35$ for values of the capital elasticity of output. Table 2 shows the values for the optimal tax rate, social welfare, and GDP growth rate when θ is changed to the value indicated, while all other parameters remain at their baseline settings.

Table 2: Effects of variations of the capital elasticity of output

		Labor income tax			Consumption tax			Sales tax		
p	θ	τ_ℓ^*	W_ℓ	γ_ℓ	τ_c^*	W_c	γ_c	τ_f^*	W_f	γ_f
0.968	0.15	0.865	476.97	313838.69	24.638	429.90	822908.92	0.850	503.03	792962.34
	0.20	0.819	325.80	5361.58	17.391	314.56	24138.71	0.800	352.03	13163.66
	0.25	0.772	236.98	489.20	13.043	244.55	2781.02	0.750	263.06	1165.73
	0.30	0.725	178.97	102.16	10.145	197.07	633.90	0.700	204.69	235.98
	0.35	0.677	138.37	34.04	8.075	162.37	212.99	0.650	163.55	76.10

Recall that because the elasticity of output with respect to public services is $1 - \theta$, when the elasticity of output with respect to capital increases, the elasticity of output with respect to public services decreases. The levels of social welfare for $\theta = 0.15, 0.20, \ldots, 0.35$ are highest when a sales tax is chosen as a source of funding for the provision of public services among the three taxation methods. Thus, even for values different from $\theta = 0.25$ set in Table 1, compared with cases in the optimal tax rates of the other taxation methods, the implication of our model that the optimal sales tax rate is the lowest and social welfare corresponding to that tax rate is the highest is invariant. Thus, the result that it is rational to use a sales tax as a funding source for the provision of public services when the government's objective is to maximize social welfare can be considered robust. Moreover, similar to the results shown in Table 1, the magnitude relationship of social welfare between the optimal labor income tax rate and the optimal consumption tax rate changes depending on the parameter value of the capital elasticity of output (θ). For example, if θ is less than 0.20, then the level of social welfare corresponding to the optimal labor income tax rate is higher. However, if θ is 0.25 or more, then the level of social welfare corresponding to the optimal consumption tax rate is higher.

In the case of $\theta = 0.15, 0.20, \ldots, 0.35$, the GDP growth rate is highest when a consumption tax is chosen as the source of funding. The reason for this result is also essentially the same as the interpretation of the result in Table 1. Specifically, when a consumption tax is adopted as the funding source of the provision of public services, the higher the consumption tax rate, the higher the GDP growth rate. Additionally, the value of the optimal consumption tax rate is much higher than the optimal tax rates of the other taxation methods. Therefore, the GDP growth rate corresponding to the optimal consumption tax rate is higher than in the case of the optimal tax rates of the other taxation methods. In all cases, the optimal tax rate, social welfare, and GDP growth rate decrease when the elasticity of output with respect to capital increases (i.e., the elasticity of output with respect to public services decreases); that is, the optimal tax rate, social welfare, and GDP growth rate are low in an economy with high capital elasticity of output. The results of the numerical analysis presented in Table 2 suggest that a small difference in the capital elasticity of output leads to a large difference in the levels of social welfare and GDP growth rate.

We analyze the effects of differences in the initial value of capital stock. In fact, there are clear differences in the initial value of the capital stock across countries (see, e.g., Penn World Table version 10.01). Therefore, it is an interesting issue to examine the implications of the model with respect to differences in the initial value of the capital stock. In this study, we consider the case of $p = 0.968$, $\theta = 0.25$, and $\sigma = 0.770$. The initial values of the capital stock change in the range from 1.0 to 5.0. Table 3 shows the results of the numerical analysis.

Table 3: Effects of variations of the initial capital stock

		Labor income tax			Consumption tax			Sales tax		
p	K_0	τ_ℓ^*	W_ℓ	γ_ℓ	τ_c^*	W_c	γ_c	τ_f^*	W_f	γ_f
0.968	1.0	0.772	236.98	489.20	13.04	244.55	2781.02	0.75	263.06	1165.73
	2.0	0.772	242.91	489.20	13.04	245.32	2781.02	0.75	268.99	1165.73
	3.0	0.772	246.38	489.20	13.04	245.77	2781.02	0.75	272.46	1165.73
	4.0	0.772	248.84	489.20	13.04	246.09	2781.02	0.75	274.92	1165.73
	5.0	0.772	250.75	489.20	13.04	246.34	2781.02	0.75	276.83	1165.73

Table 3 shows that differences in the initial value of capital stock do not affect all optimal tax rates and GDP growth rates. However, differences in the level of social welfare corresponding to the optimal tax rates of the three taxation methods arise depending on the initial value of the capital stock. As with the cases in Tables 1 and 2, from the results shown in Table 3, the level of social welfare is highest in cases in which a sales tax is adopted. Thus, when the objective of the government is to maximize social welfare, it is desirable to adopt a sales tax as the source of funding for the provision of public services. This result can be interpreted as robust in the sense that the implications of the model are invariant, along with the implications of the model examined based on Tables 1 and 2.

As in the cases in Tables 1 and 2, the magnitude relationship between the level of social welfare corresponding to the optimal labor income tax rate and the level of social welfare corresponding to the optimal consumption tax rate is not definitive. Specifically, Table 3 suggests that in economies with relatively small initial capital stocks, the level of social welfare in the optimal consumption tax rate case is higher, whereas in economies with relatively large initial capital stocks, the level of social welfare in the optimal labor income tax rate case tends to be higher. Although this is an expected result, moreover, for all taxation methods, the larger the initial value of capital stock, the higher the level of social welfare; that is, an economy with a higher level of initial capital stock starts with a higher level of social welfare. Furthermore, the result that the GDP growth rate is the highest in the case in which a consumption tax is used as a financial source for public service provision compared with cases with the other taxes is similar to the results in Tables 1 and 2.

Thus far, we have assumed that the purpose of the government is to maximize social welfare. However, as indicated by Misch et al. (2013) from a practical point of view, because governments typically encounter informational constraints, such as household preferences, and social welfare is more difficult to measure than output, governments tend to seek output growth maximization. Therefore, policy-making authorities may set a target level for the GDP growth rate in some cases. As an illustrative example, China's central government set the target GDP growth rate to around 5% for 2024 at the 14th National People's Congress in March 2024 (see, `https://www.cnbc.com/2024/03/05/china-gdp-target-2024.html`). In view of the above consideration, in the following discussion, like Watanabe *et al.* (2015), we assume a scenario in which the government sets a target level for the GDP growth rate. In particular, we are interested in the long-term slump in macroeconomic performance in many developed countries, which some economists call "secular stagnation" (see, e.g., Summers, 2014; Fritz *et al.*, 2019; Rachel and Summers, 2019; Hamada and Sakurai, 2022; Podkaminer, 2022; Strauss and Yang, 2024). [4] Thus, it would be a meaningful approach to examine

[4] As for the case of Japan, the period of very low economic growth performance between 1992 and 2012 is

macroeconomic policy issues, given the scenario of the normalization of low economic growth. In this regard, the research of Jorgenson and Vu (2021) provides a useful reference point in our numerical analysis. Jorgenson and Vu (2021) estimated the economic growth rates of G7 member countries during the period between 2018 and 2028, and reported an average annual growth rate of 0.91% in the most pessimistic case, 1.18% in the base case, and 1.46% in the most optimistic case. In this study, we use the growth projections of Jorgenson and Vu (2021) for G7 member countries and analyze cases in which the government sets target annual GDP growth rates of 0.91%, 1.18%, and 1.46%. Because our OLG model uses 30 years as one period, the adjusted gross GDP growth rates in one period corresponding to the annual rates of 0.91%, 1.18%, and 1.46% are 1.312, 1.422, and 1.545, respectively. The parameter θ is considered as the baseline setting where $\theta = 0.25$. Table 4 presents the tax rate of each taxation method and social welfare corresponding to each target level of the GDP growth rates.

Table 4: Tax rate and social welfare corresponding to the target level of the GDP growth rate

Growth	Gross growth			Labor income tax		Consumption tax		Sales tax	
(annual rate)	(period adjusted)	p	σ	τ_ℓ	W_ℓ	τ_c	W_c	τ_f	W_f
0.91%	1.312	0.968	0.847	0.067	22.89	0.078	21.27	0.050	22.49
			0.770		10.77		9.63		10.47
			0.693		6.43		5.51		6.18
1.18%	1.422	0.968	0.847	0.069	29.37	0.080	27.29	0.051	28.96
			0.770		13.77		12.31		13.47
			0.693		8.21		7.04		7.96
1.46%	1.545	0.968	0.847	0.071	36.04	0.083	33.48	0.053	35.62
			0.770		16.87		15.07		16.56
			0.693		10.04		8.62		9.78

From the results presented in Table 4, among a labor income tax, consumption tax, and sales tax as a source of funding for public service provision, the sales tax rate is found to be the lowest, given the condition by which the government sets a target level for the GDP growth rate of less than around 1.5% per annum. However, the level of social welfare is higher in the case in which a labor income tax is selected than the case in which a consumption tax or

often referred to as the "the lost two decades" (for details, see Ito and Hoshi, 2020). Moreover, the prolonged state of low economic growth in many advanced economies (i.e., the normalization of low growth) may be viewed as a kind of hysteresis. In this regard, Commendatore *et al.* (2011), Mendieta-Muñoz (2017), and Dell'Erba *et al.* (2018) provide a useful discussion.

sales tax is selected as the source of finance for the provision of public services.[5] This implies that a labor income tax is more desirable than a consumption tax or sales tax for funding the provision of public services in a scenario in which the government sets the target value of a low GDP growth rate at less than around 1.5% per annum. Note that when comparing the cases of a sales tax and consumption tax, the level of social welfare in the case in which a sales tax is adopted is larger. Additionally, although the level of social welfare corresponding to a sales tax is lower than that corresponding to a labor income tax, the difference in social welfare between the two is not very large. In terms of the GDP growth rate, as the target level for the GDP growth rate increases, the tax rates for all taxation methods also increase accordingly. Furthermore, regardless of the taxation method, the more the government emphasizes the utility of future generations (i.e., larger values of σ), the higher the level of social welfare.

In the following analysis, we focus on the case in which the target annual GDP growth rate is 1.18%, which is the base case for the growth projections of G7 member countries used by Jorgenson and Vu (2021). In the present study, prior to examining the effects of the progress of population aging on the tax rate of each taxation method and social welfare, we consider actual data on the ratio of the population aged 65 and over to the total population and life expectancy at birth. These data are available from the World Bank's World Development Indicators (refer to `https://databank.worldbank.org/source/world-development-indicators`). Figure 1 shows the change in the ratio of the population aged 65 and over to the total population (i.e., the rate of the population of older adults) from 1960 to 2022 for G7 member countries.

From Figure 1, at first glance, we find that the rates of the population of older adults in all G7 member countries have been increasing consistently. In particular, the level of Japan's population aging rate is significantly higher than that of the other G7 member countries. The World Health Organization and the United Nations define an aging society as one in which the percentage of the total population aged 65 or older exceeds 7%, an aged society as one in which this percentage exceeds 14%, and a super-aged society as one in which this percentage exceeds 21% (see Tahara, 2016). In 2022, the year of the most recent data available, because France, Germany, Italy, and Japan, among G7 member countries, have the ratio of the population aged 65 and over to the total population exceeding 21%, these countries are regarded as super-aged societies. It should be noted that, as of 2006, Japan had already exceeded 21%, making it the fastest country among G7 member countries to become a super-aged society.

[5] Watanabe *et al.* (2015) described the reason that the level of social welfare is higher in the case in which a labor income tax is adopted than in the case in which a consumption tax is adopted as follows: When the target level of GDP growth rate is low, the future generation's utility is low. However, a consumption tax reduces the utility of older people. By contrast, a labor income tax does not reduce the utility of older people in the initial period but does reduce the future generation's utility. As a result, the level of social welfare in the labor income tax case is relatively high.

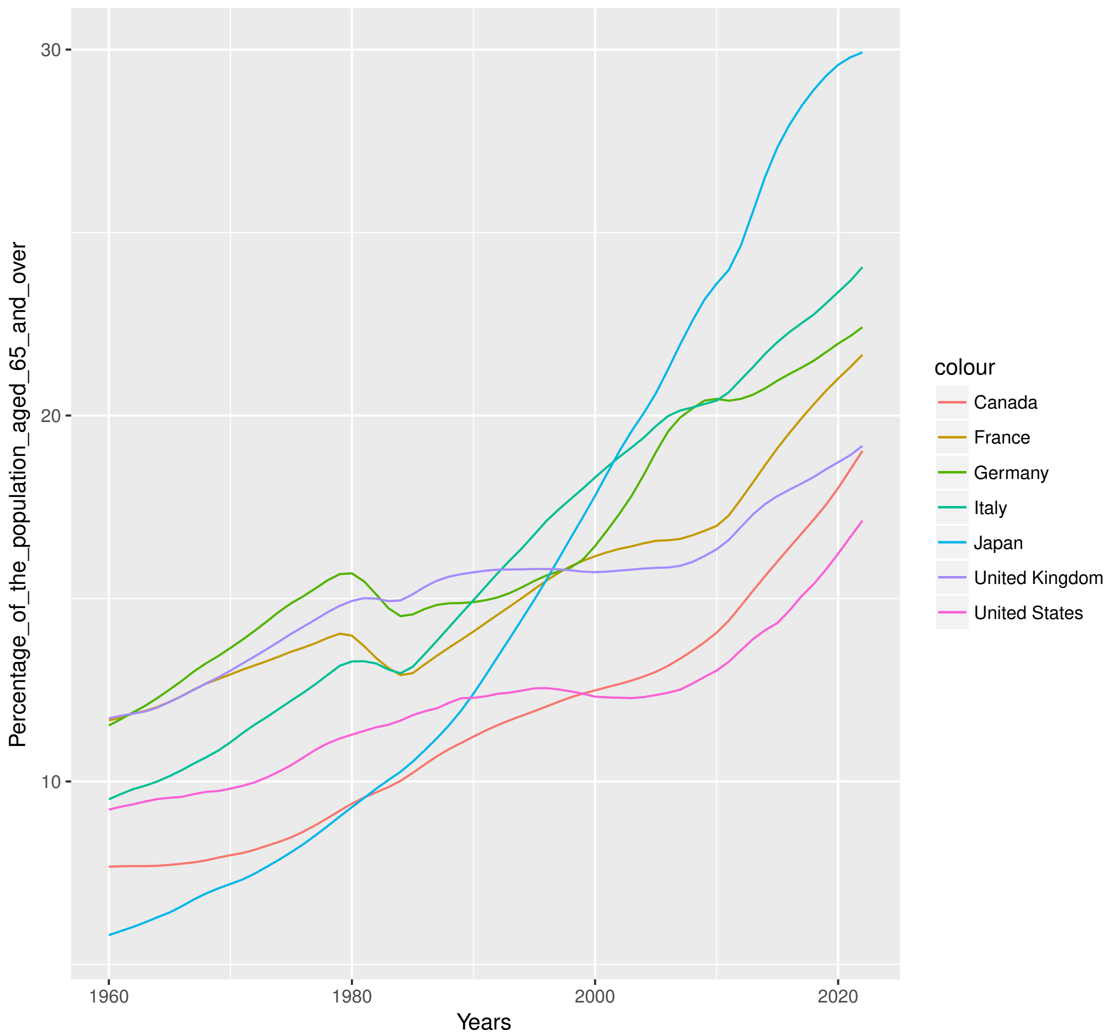

Figure 1: Ratio of the population aged 65 and over to the total population (%) in G7 member countries, 1960–2022

Additionally, the number of years required to reach an aged society from an aging society is called the doubling years, and Japan's doubling years were only 25 years (see Komine and Kabe, 2009).

Figure 2 shows a graph of life expectancy at birth in G7 member countries for the period from 1960 to 2022.

Observing the graph for each country in Figure 2, there exist broad upward trends in life expectancy at birth for all G7 member countries within the time period of the data. Again, Japan has the highest level of life expectancy at birth among G7 member countries. Note that, since 2020, life expectancy at birth in G7 member countries has experienced a slight decline, although its degree varies by country. Incidentally, it is still fresh in our minds that

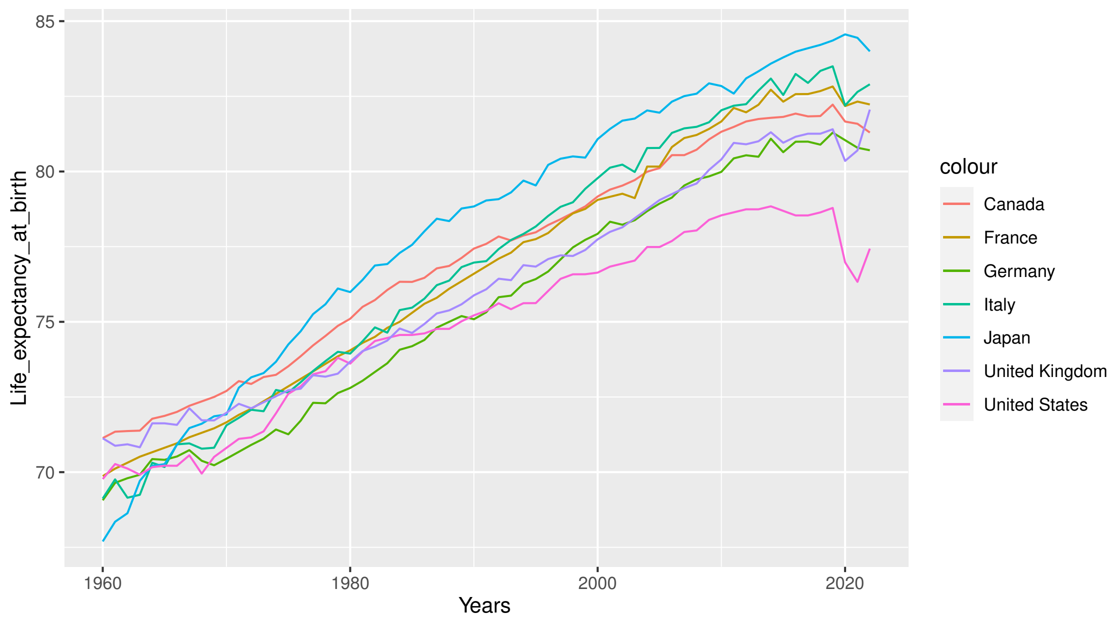

Figure 2: Life expectancy at birth in G7 member countries, 1960–2022

COVID-19 occurred in Wuhan City, Hubei Province, China in December 2019, and spread to almost all countries in the world in March 2020. Thus, a temporary decline in life expectancy at birth in G7 member countries after 2020 may be attributable to COVID-19. From the above consideration regarding data on the rate of the population of older adults and life expectancy at birth, we confirm the evidence that population aging is steadily progressing in G7 member countries.

Now, based on the above evidence, we conduct numerical analysis with respect to the effects of population aging. For the parameters σ and θ, we consider the case of $\sigma = 0.770$ together with the baseline setting of $\theta = 0.25$. We now explain the rationale for examining the variation in parameter p in our quantitative analysis with respect to the effects of population aging. In our model, parameter p denotes the probability of survival into the old-age period; hence, an increase in p can be interpreted as an improvement in life expectancy at birth. Moreover, the rate of the population of older adults at each point in time is represented by $p/(1+p)$. Therefore, an improvement in life expectancy at birth results in an increase in the rate of the population of older adults because an increase in p leads to an increase in $p/(1+p)$. Evidently, this relationship between life expectancy at birth and the rate of the population of older adults is consistent with the actual data shown in Figures 1 and 2. Table 5 presents the tax rate of each taxation method and social welfare when parameter p is changed to the value indicated in the range $[0.60, 0.95]$, while all other parameters remain at their baseline settings.

Table 5: Tax rate and social welfare associated with the progress of population aging

Growth		Labor income tax		Consumption tax		Sales tax	
(annual rate)	p	τ_ℓ	W_ℓ	τ_c	W_c	τ_f	W_f
1.18%	0.60	0.0764	15.69	0.0789	12.87	0.0569	15.48
	0.65	0.0750	15.43	0.0789	12.74	0.0559	15.21
	0.70	0.0738	15.17	0.0789	12.63	0.0550	14.94
	0.75	0.0727	14.91	0.0790	12.54	0.0541	14.67
	0.80	0.0717	14.65	0.0792	12.46	0.0534	14.40
	0.85	0.0708	14.39	0.0795	12.40	0.0527	14.12
	0.90	0.0700	14.13	0.0800	12.36	0.0521	13.85
	0.95	0.0692	13.87	0.0801	12.32	0.0516	13.57

The results in Table 5 show that if the target level of the GDP growth rate is set to 1.18% per annum, when population aging progresses, the labor income tax and sales tax rates decrease, whereas the consumption tax rate conversely increases. Moreover, for all values of p, the level of social welfare is highest in the case in which a labor income tax is used to finance public service provision, although the tax rate is lowest in the case in which a sales tax is used. Therefore, given the target GDP growth rate of 1.18% per annum and in a scenario in which population aging inevitably progresses, it is reasonable to adopt a labor income tax as a source of funding for the provision of public services. Furthermore, with steady population aging, the level of social welfare declines for all taxation methods; that is, under the scenario of low growth performance in which the target level for the GDP growth rate is 1.18% per annum, the progress of population aging has a negative influence on social welfare for all taxation methods. We find that these are different results from those presented in Table 1.

Note the following points. Table 1 shows the results in a scenario in which the GDP growth rate and social welfare corresponding to the socially optimal tax rate are achieved for each taxation method. By contrast, Table 5 presents the results with respect to the tax rate of each taxation method and social welfare under the given target GDP growth rate of 1.18% per annum. The differences in the results for the relationship between population aging and social welfare in Tables 1 and 5 can be summarized as follows: The case of Table 1 indicates that there is a positive correlation between population aging and social welfare in a scenario in which GDP growth related to the socially optimal tax rate is achieved. However, in the case of Table 5, the results imply that the level of social welfare declines in line with the aging of the population under a normalized low GDP growth rate, which may be regarded as a type of secular stagnation. Additionally, with respect to the progress of population aging indicated in Table 5, the level of social welfare is relatively higher in the case of a labor income tax than in the case of the other taxation methods.

The major implications of our model obtained from Table 5 can be interpreted as follows: Regarding a labor income tax and sales tax, given a target GDP growth rate, Eqs. (17) and (32) imply that the labor income tax rate, τ_ℓ, and sales tax rate, τ_f, decrease when life expectancy, p, increases. Moreover, Eqs. (19) and (33) imply that an increase in life expectancy, p, has both positive and negative effects on social welfare, given a specific target GDP growth rate. The positive effect is that an increase in household consumption caused by an increase in p results in a higher level of social welfare. The negative effect is that, because of a decrease in earnings from the annuitized savings received in the old-age period when p increases, an increase in p results in a decline in the level of social welfare. In this regard, given p, households emphasizing their future consumption and raising their subjective discount factor would be expected to have a negative effect on social welfare.

Regarding a consumption tax, given a target GDP growth rate, Eq. (23) means that if life expectancy, p, increases, then the consumption tax rate, τ_c, must increase. Moreover, in the case of a consumption tax, as in the case of a labor income tax and sales tax, Eq. (25) indicates that an increase in life expectancy, p, has positive and negative effects on the level of social welfare. In particular, as mentioned above, because the consumption tax rate, τ_c, increases as life expectancy, p, increases, the negative effect caused by such an increase in the consumption tax rate, τ_c, on social welfare outweighs the positive effects in Eq. (25). Thus, this results in a decline in the level of net social welfare as life expectancy increases. To summarize, the results presented in Table 5 suggest that the negative effects exceed the positive effects on social welfare for all taxation methods. Consequently, the level of social welfare declines along with the progress of population aging.

Finally, we quantitatively analyze the effects of technological progress at the economy-wide level in a country on the tax rate of each taxation method and social welfare in the case of a target annual GDP growth rate of 1.18%. For the parameter variation of the technological level, A, we assume the values $13.7, 13, 8, 13, 9, \ldots, 14.6$. Additionally, we focus on the case of $p = 0.968$ and $\sigma = 0.770$ among the baseline settings. Table 6 shows the tax rate of each taxation method and social welfare when parameter A is changed to the value indicated, whereas all other parameters are held constant at the baseline specifications.

The results presented in Table 6 are summarized as follows: First, we examine the effects of technological progress on the tax rates of the three taxation methods. The results presented in Table 6 show that the tax rates for all methods decline with technological progress, regardless of whether a labor income tax, a consumption tax, or a sales tax is adopted to finance the provision of public services. Moreover, the tax rate is the lowest among three taxation methods when a sales tax is adopted as a financial source for the provision of public services. Next, we

Table 6: Tax rate and social welfare associated with technological progress

Growth		Labor income tax		Consumption tax		Sales tax	
(annual rate)	A	τ_ℓ	W_ℓ	τ_c	W_c	τ_f	W_f
1.18%	13.7	0.0684	13.77	0.0796	12.31	0.0510	13.47
	13.8	0.0678	13.76	0.0788	12.32	0.0505	13.47
	13.9	0.0671	13.76	0.0780	12.33	0.0500	13.47
	14.0	0.0665	13.76	0.0772	12.33	0.0495	13.47
	14.1	0.0658	13.76	0.0764	12.34	0.0491	13.47
	14.2	0.0652	13.75	0.0756	12.34	0.0486	13.47
	14.3	0.0646	13.75	0.0749	12.35	0.0481	13.47
	14.4	0.0639	13.75	0.0741	12.36	0.0477	13.47
	14.5	0.0633	13.74	0.0734	12.36	0.0472	13.47
	14.6	0.0628	13.74	0.0727	12.37	0.0468	13.47

investigate the impact of technological progress on social welfare. Table 6 indicates that the relationship between technological progress and social welfare depends on which of the three taxation methods is adopted to finance the provision of public services. Specifically, social welfare declines with technological progress when a labor income tax is adopted, whereas social welfare increases with technological progress when a consumption tax is adopted. The level of social welfare corresponding to the case in which a sales tax is adopted is independent of the level of technology and remains constant. Furthermore, among the three taxation methods, the level of social welfare is the highest when the labor income tax is adopted to finance the provision of public services. Therefore, under the condition that the target GDP growth rate is 1.18% per annum, the use of a labor income tax as a source of the provision of public services can be seen as reasonable in countries in which technological progress is observed at the economy-wide level in terms of the level of social welfare.

5 Conclusion

In this study, we constructed a theoretical framework based on incorporating a sales tax into the OLG model presented by Watanabe *et al.* (2015) to conduct numerical analyses based on an extended OLG model. Specifically, under several perspectives different from those of Watanabe et al. (2015), we examined which option from a labor income tax, a consumption tax, or a sales tax should be used as the source of funding for the provision of productive government expenditure (i.e., productive public services) using a computable OLG model. It should be noted that, although Watanabe *et al.* (2015) discussed aging, to some extent, they did not conduct a detailed analysis with respect to issues of the influence of the progress of population aging. By contrast, we analyzed the impact of a steady increase in population aging on the tax rate of each taxation method and social welfare in detail from a quantitative perspective. Moreover, we numerically analyzed the consequences of differences in the capital elasticity of output, in addition to the initial value of capital stock on social welfare and the GDP growth rate corresponding to the optimal tax rates of the three taxation methods. Furthermore, we considered the effects of technological progress on the tax rates of the three taxation methods and social welfare under the low target GDP growth rate of 1.18% per annum. Watanabe *et al.* (2015) did not address these issues.

The significant contribution of this study from a theoretical perspective is the derivation of meaningful implications that complement the analytical results presented by Watanabe *et al.* (2015) in the context of our extended model with more taxation options. Moreover, by comparing the scenario in which the government aims to maximize social welfare with the scenario in which the government set a target GDP growth rate of less than around 1.5% per annum (i.e., normalized low growth performance), our quantitative approach provides new insights into the relationship between the steady progress of population aging and changes in social welfare. This is the most important contribution of our study. Another notable contribution is the presentation of new findings obtained through introducing a sales tax as an increase in the number of options regarding the fiscal issue of which taxation method is the most reasonable approach to finance the provision of productive public services; that is, we contributed to a better understanding of which taxation method is reasonable to adopt as a means of financing public services, which is considered as a crucial issue in macroeconomic policy.

Since the sustainable development goals (SDGs) were adopted by the United Nations General Assembly in September 2015, interdisciplinary research on the SDGs has been actively conducted. Under the current circumstances, however, the majority of earlier studies on the SDGs have been analyzed based on empirical approaches. By contrast, very few cases exist of analysis from the viewpoints of theoretical approaches (a notable exception is Noda and Kano, 2021). Our study is particularly related to the content of goal 8 (target 8.1) in the SDGs ("Decent Work and Economic Growth"). Clearly, this SDG target is closely related to economic growth. However, to the best of our knowledge, there are no studies in which researchers have discussed the context of models of economic growth regarding goal 8 (target 8.1) of the SDGs. Therefore, this study can be regarded as a first step in a theoretical approach to research on goal 8 (target 8.1) of the SDGs. Additionally, the analytical framework based on our modeling provides a theoretical foundation for this SDG target. Specifically, we can present useful suggestions to enhance the standard of living of people in developing and developed countries through findings based on our OLG model for achieving the maximum growth rate of GDP per capita by selecting the taxation method. This is an important contribution in terms of the interdisciplinary aspect of this study.

The major results can be summarized as follows: When the government's objective is to maximize social welfare, it is rational to opt for a sales tax as a source of financing for the provision of productive public services. This result is a new finding that cannot be found in the study by Watanabe *et al.* (2015), who considered only a labor income tax and a consumption tax. Moreover, we obtained the following new findings. Focusing on differences in the capital elasticity of output, the optimal tax rates of all taxation methods, social welfare, and GDP growth rate decrease as the capital elasticity of output increases. This implies that the optimal tax rate, social welfare, and GDP growth rate are relatively high in an economy with a low capital elasticity of output. Furthermore, although differences in the initial value of capital stock do not affect all types of optimal tax rates and GDP growth rates, the level of social welfare is highest in the case in which a sales tax is adopted compared with the cases of the other taxation methods. Next, from a different perspective from the analysis above, we considered a scenario in which the government sets a target level for the GDP growth rate in view of the recent normalized low economic growth performance facing many developed countries. Specifically, we analyzed the relationship between the tax rate of each taxation method and social welfare based on the economic growth outlook for G7 member countries for the period between 2018 and 2028 provided by Jorgenson and Vu (2021). As a result, social welfare is higher when a labor income tax is selected to fund the provision of productive public services than when a consumption tax or sales tax is selected, given the condition under which the government sets a target level for the GDP growth rate of less than 1.5% per annum. This suggests that a labor income tax is more desirable in a low-growth economy as a source of funding for the provision of productive public services. Moreover, we found that social welfare is highest when a labor income tax is used as a source of funding for the provision of productive public services in the case of steadily aging population, given the low target GDP growth rate of 1.18% per annum. This implies that under the circumstances of the steady progress of population aging, in addition to the the normalization of low GDP growth rate, a labor income tax offers advantages over the other taxation methods. Regarding the taxation methods, when population aging is progressing, a labor income tax and sales tax decrease, whereas a consumption tax rate increases. The impacts of population aging on social welfare and tax rates among various taxation methods as described above were not analyzed by Watanabe *et al.* (2015); hence, these are new findings of our study. Furthermore, technological progress at an economy-wide level has the effect of reducing the tax rates of the three taxation methods (i.e., a labor income tax, a consumption tax, and a sales tax) as financial sources for public service provision.

Finally, we mention several directions for the expansion of our research. In this study, we considered productive public services as a flow variable. Thus, a comparative analysis with the case of public capital as a stock variable, such as Futagami *et al.* (1993), would be a meaningful research direction. Additionally, infrastructure obsolescence has become an increasing problem in many developed countries in recent years. Therefore, similar to Noda and Osano (2017), it would be worthwhile for us to construct a model that incorporates the maintenance of public capital stock and analyze the effects of infrastructure life-extension policies corresponding to various taxation methods. Furthermore, in reality, pollutants are emitted as by-products of corporate production activities, and the issue of environmental conservation is considered to be as important as the issue of population aging, which we addressed in this study. For example, as discussed by Noda and Kano (2021), let us assume an economy in which a zero-emissions policy for reducing the net flow of pollution to zero is implemented in response to a growing social awareness of environmental issues. Under such a scenario, we can expect to gain new insights into the issue of the selection of taxation methods by examining the sources of public service provision that maximize the level of social welfare. These points will be the topics of future research.

Acknowledgements

This study was funded in part by a Grant-in-Aid for Scientific Research (C) (20K01639) from the Japan Society for the Promotion of Science.

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